A model of voting with motivated reasoners: with applications to affect polarization

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What is the optimal policy response to an angry voter? It has been extensively documented that voters' positive and negative feelings about parties and candidates shape how they place them on the ideological space. This is the result of motivated reasoning, where voters seek out information that aligns with their "likes" and "dislikes" and use this information to infer the position of parties. In this article, we leverage these well documented biases to describe the effect of non-policy shocks, such as an increase in affect polarization or a COVID-19 diagnostic, on the ideological placement of parties. The proposed model is extremely general and accommodates known survey evidence on perceptual bias. We exemplify its usefulness analyzing 50 years of presidential elections in the US and by reanalyzing experimental data on negative social media messages.

Ideology | Perceptual Bias | Assimilation and Contrast | Polarization

An ever expanding literature shows that "partisanship in American politics is inextricably linked with social identities."(Mason et al., 2021) These partisan identities "color one's interpretation of political information"(Bolsen et al., 2014, pp.236), including our perceptions of the out-group voters as well as the relative liberal-conservative leaning of the party Ahler and Sood (2018). Increasingly, the literature describes voters as "motivated reasoners who seek out congenial sources of information and defend their attitudes and beliefs when challenged"(Strickland et al., 2011, pp. 935). In a series of landmark studies, Lodge and Taber (2013) demonstrate that "motivated reasoning -the systematic biasing of judgements in favor of automatically activated, affectively congruent beliefs and feelings- is built into the basic architecture and information processing of the brain"(Lodge and Taber, 2013, pp 24).

The centrality of identity politics and motivated reasoning has led some scholars to discount the value of labels such as "ideology", "left-right", "progressive", or "conservative", in favor of conceptualizations that rest on party affinity (Levendusky, 2009; Mason et al., 2021; Mason, 2016). However, decades of survey data of American voters shows that Democrats and Republicans still hold stable and well defined locations on the reported left-right or progressive-conservative scales.[1] In other words, party identities, affect, as well as negative and positive valence have some underlying translation to reported locations on the ideological space.[2] A voter angered by a Democratic or Republican candidate or event will project this anger to actual locations on the ideological space. In this article, we model these affective shocks.

Our objective in this article is to integrate existing findings on motivated reasoning into the classic downsian model of party vote. We describe how perceptual biases alter the ideological location of parties as reported by voters.[3] These perceptual biases are also known as ***assimilation*** and ***contrast*** effects, with voters perceiving parties they support as ideologically closer than their "true" location (*assimilation*) and parties they do not support as being further removed (*contrast*).

Our model leverages *assimilation* and *contrast* effects to explain the pressure exerted by motivated reasoners on the ideology advertised by parties. The problem was previously addressed in the work of Adams, Merrill, and Grofman (2005), who hypothesized that *assimilation* and *contrast* were responsible for the attenuation biases in their equilibrium results. The solution proposed by Adams et al. (2005), however, required researchers to arbitrarily decide a "party policy preference" to counter the observed attenuation bias in their valence model of voting.[4] Different from AMG, the model we propose in

[1] See West and Iyengar 2020 for a discussion of the relationship between partisanship and issue cleavages.

[2] Aldrich and McKelvey (1977) formalize this intuition: "we feel that operationalizations of tests of the spatial model have to be based on perceptual, as opposed to preference data" (Aldrich and McKelvey, 1977, pp.111). Their model excels in correcting for bias under the assumption that "candidates occupy fixed positions in an issue space and that the individual perceptual data arises from this via a two-step process, the first step consisting of 'true' error in perception, and the second step consisting of distortion introduced in the actual survey situation" (Aldrich and McKelvey, 1977, pp.112). In our analysis, however, motivated reasoning will have different effects among supporters and opponents of a party (mixture). Further, voters who switch parties will move from one group (assimilated) to another (contrasted). Therefore, errors will not normally distributed around a 'true' location. In fact, we argue, rather than a true location we may model a "center" of the ideological space that serves as anchoring.

[3] The extant literature uses a number of different terms to describe misperceptions in the location of parties, including *perceptual bias* (Aldrich and McKelvey, 1977; Gerber and Green, 1999; Aldrich et al., 2018), *projection bias* (Muraoka and Rosas, 2020), *halo effect* (Kahneman, 2011), and *assimilation and contrast* (Adams et al., 2005). We will restrict the terminology of our article to *perceptual bias* for the general effect and to *assimilation* and *contrast* for the positive and negative perceptual biases respectively. A strict definition of perceptual bias, assimilation, and contrast are in the introduction of the Supplemental Information File (SIF)

[4] See Chapter 10 in Adams, Merrill, and Grofman (2005).

Significance Statement

Voters' positive and negative feelings about parties and candidates shape how they place them on the policy space. This is the result of motivated reasoning, where voters seek out information that aligns with their "likes" and "dislikes" and use this information to infer the position of parties. In survey data, these type of perceptual bias is known as ***assimilation*** and ***contrast***, with supporters perceiving parties to be ideologically closer (assimilation) and detractors perceiving the same parties further removed (contrast) from their mean location in a sample. In this article, we present a model that leverages *assimilation* and *contrast* effects to explicitly account for motivated reasoning. The model allows scholars understand the effect of negative or positive valence and affective shocks on the reported ideological location of parties.

this paper does not require fine tuning and focuses instead on the mechanics of party change under positive and negative perceptual biases. We demonstrate that, as parties move to the left or right of the political spectrum, the ideological space changes and the "center" moves.

In the basic downsian model, among other key conditions summarized by Grofman (2004), a citizen is expected to cast her vote for the party or candidate that is ideologically closest.[5] However, we posit, voters compare the "assimilated" position of the party they like to the "contrasted" position of the party they do not like. Once we replace the "true" location of the party by its perceived location, a number of interesting consequences follow.

Our model reports a ***surprising finding*** with broad implications for how we describe the optimal location of the parties: **under motivated reasoning there is no fixed "center" of the ideological space.**[6] Indeed, the location where two parties are observed as identical (the "center") will vary for voters of the different parties.[7] If Democratic voters see the parties as identical (i.e. at the "center"), Republican voters will not.[8] If Republican voters see the parties as identical, Democratic voters will not. Therefore, there is no convergence by both parties to a common center of the ideological space. There are only centripetal and centrifugal movements observed by the voters of both parties. The stretching and compressing of the ideological space, to use Aldrich and McKelvey's description, often cancels perceived centripetal and centrifugal changes by parties.

Explicit modeling of *assimilation* and *contrast* allows us to describe the expected consequences of affect polarization on ideological polarization. As we leverage these well documented biases, we are able to translate non-directional effects into directional ones. We explore how increases in asymmetric affect polarization ("likes" and "dislikes" that are more significant among some voters) yield asymmetric optimal ideological offers. We describe the substantive relevance of our model using two different examples: first, we use data from the *National Election Study* over 50 years to model changes in the parties locations and the policy space during presidential elections. Second, we reanalyze survey experiments on negative social media messages to measure how the location of the parties (and the center of the policy space) are modified by negative social media treatments.

The article is organized as follows: in the first section we provide a brief introduction to assimilation and contrast as it is described in the extant literature. In the second section we describe how a motivated downsian voter would perceive a change in the ideological positions of parties. Specifically, we show how changes in 'likes' and 'dislikes' alter the perceived centripetal and centrifugal movement of candidates and parties in the ideological space and, as important, how this space changes. In the third section we exemplify the substantive implications that we derive from the model using the National Election Study in the United States. In the fourth section we reanalize the results of a survey experiment that uses negative social media messages to measure changes in the perceive location of parties. We conclude in the fifth section.

1. The Motivated Downsian Voter

Downsians models[9] of party vote begin with an individual who compares the ideological location of the candidates (and parties) to her own. In standard downsian models, a voter with ideal point x_i observes the "true" location of parties with ideology L_k and uses this information to vote for the one that is ideologically closest, $(x_i - L_k)^2 - (x_i - L_{\neg k})^2 > 0$.

Despite its elegance and simplicity, Downsian models of party vote have troubled scholars for decades, as voters often care about other non-directional policy offers (Stokes, 1963; Schofield and Sened, 2005) and evidence mounts that voters seldom observe L_k and $L_{\neg k}$ when deciding their vote (Adams et al., 2005; Aldrich and McKelvey, 1977). Instead, voters observe a projection of the party's location that is shifted by their "likes", and "dislikes", the issues being evaluated, their socio-demographic background among a large number of other voter characteristics (Aldrich et al., 2018). We define this *motivated Downsian voter* as an individual who observes an assimilated, $A_k(x) = \alpha_k^A + \beta_k^A x$, or a contrasted, $C_k(x) = \alpha_k^C + \beta_k^C x$, position of the party rather than its "true" location, where x is the voter's ideal point.

Assimilation and *contrast* are prevalent in survey data. Merrill et al. (2001) analyze elections in Norway, France, and the United States and describe consistent assimilation and contrast effects. As they estimate the magnitude of the perceptual bias, they evaluate distortions in the random placement of parties around a true mean (stochastic variation) as well as distortions in the ideological space (anchoring effects). location of the parties. Calvo, Hellwig, Kiyoung (2014) study 83 parties in 13 countries of the Comparative Study of Electoral Systems (CSES) and find large perceptual biases in 82 of them. Both Aldrich et al. (2018) and Muraoka and Rosas (2020) analyze individual, economic, and party system determinants of perceptual bias in over 300 parties, showing higher *deviations* in older democracies, majoritarian electoral systems with presidential elections. In the United States, one of those older, majoritarian, and presidential regimes, the assimilation and contrast curves are extraordinarily stable over 50 years of the National Election Study data (ANES, 2019).[10]

A visual description of *assimilation* and *contrast*. Figure 1 provides a first glance of assimilation and contrast in current survey data. The horizontal axis describes the self-reported

[5] Bernard Grofman (2004) presents a wonderful account of the different requirements for Downsian convergence in a two party system. We are single mindedly focusing on one of the challenges (i.e. number 8 in Grofman), which assumes that voters correctly identify the true location of the parties in the policy space.

[6] We define the "center" of the ideological space as the location where two parties would be observed as identical to each other. As we will show, this location is unlikely to be the same for Democratic and Republican voters. In fact, between 1972 and 2020 there is no election in which the "center" of the ideological space was the same for voters of both parties.

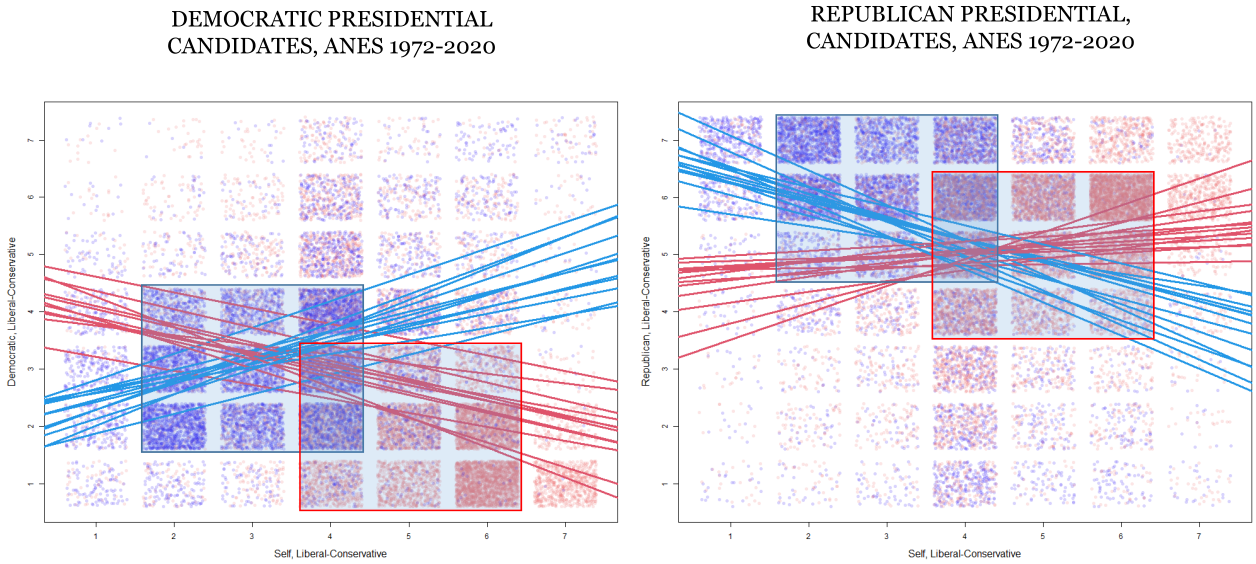
[7] This is one of the two critical areas in which our model differs from Aldrich and McKelvey (1977). As a latent model problem, Aldrich and McKelvey (1977) scaling strategy seeks to uncover the "true" location of the parties once distortions in the ideological space are accounted for. This perceived location is the same for the assimilated and contrasted group and so is the "center" of the ideological space. In our analysis, neither the locations of the parties nor that of the ideological center is the same for the two groups of voters.

[8] See the appendix for a derivation of the global "center". This global center is trivial for the equilibrium position of parties and, for all practical purposes, is also trivial to voters, given that each of them observes the center in a position that is not this global center.

[9] A technical description of the model is provided in the online Supplemental Information File (SIF). Here we present a more accessible summary for the general reader.

[10] In the early 1970s and 1980s there are a number of studies that documented the prevalence of assimilation and contrast in US politics. Interestingly, most survey data showed large assimilation and modest contrast effects, as documented in the metanalisis of Granberg et al. (1981). Since the 1990s, on the other hand, assimilation has declined and contrast has increased in importance. In their 1981 article, Granberg et al. (1981) noted: "It is still not known why a candidate is assimilated or contrasted to different degrees on different issues." After the 1980s, despite how well documented are these effects, research interest declined. We believe that was premature and that modeling perceptual bias explicitely is an important research agenda

Fig. 1. Assimilation and Contrast in the United States from 1972 through 2020



Note: The horizontal axis describes the self-reported location of the voters. The vertical axis describes the reported location of all Democratic presidential candidates (left) and all Republican presidential candidates (right). Points jittered to facilitate visualization. Democratic voters represented with blue solid dots and Republican voters represented with red solid dots. Transparent squares show that Democratic voters perceived the Democratic party as moderate while Republicans perceive it as extreme left. Similarly, Republican voters perceive the Republican party as moderate while Democratic voters perceive it as extreme right. Assimilation and contrast lines estimated for each presidential candidate. Upward lines describe assimilation while downward lines describes contrast. Full figures for each of the presidential candidates are in the appendix. Our own estimates from ANES (2019).

location of the voters. The vertical axis describes the reported location of Democratic presidential candidates (left) and Republican presidential candidates (right), with Democratic voters represented using solid blue dots and Republican voters represented with solid red dots. As shown by the Transparent squares,that Democratic voters perceive the Democratic party as moderate while Republicans perceive the same party as extreme left. Similarly, Republican voters perceive the Republican party as moderate while Democratic voters perceive the same party as extreme right. The Assimilation lines show that as voters become more conservative so does the perceived location of both parties. The contrast lines, on the other hand, show that as voters become more conservative the party is perceived as more liberal. Therefore, upward lines describe assimilation while downward lines describes contrast. Notice that Figure 1 summarizes the available information in the ANES data ANES (2019) and requires no assumptions.[11] However, as we will show next, there is considerable more information that may be explored using this data.

Figure 2 describes significant new information that may be extracted from the existing survey data. It describes the relationship between the self-reported placement of respondents on a liberal-conservative dimension (x-axis) and the placement of Donald Trump (y-axis). The linear *assimilation* estimates follow from:[12]

$$A_R(x) = \alpha_R^A + \beta_R^A x, R == 1$$

The *contrast* estimates follow from:

$$C_R(x) = \alpha_R^C + \beta_R^C x, R == 0$$

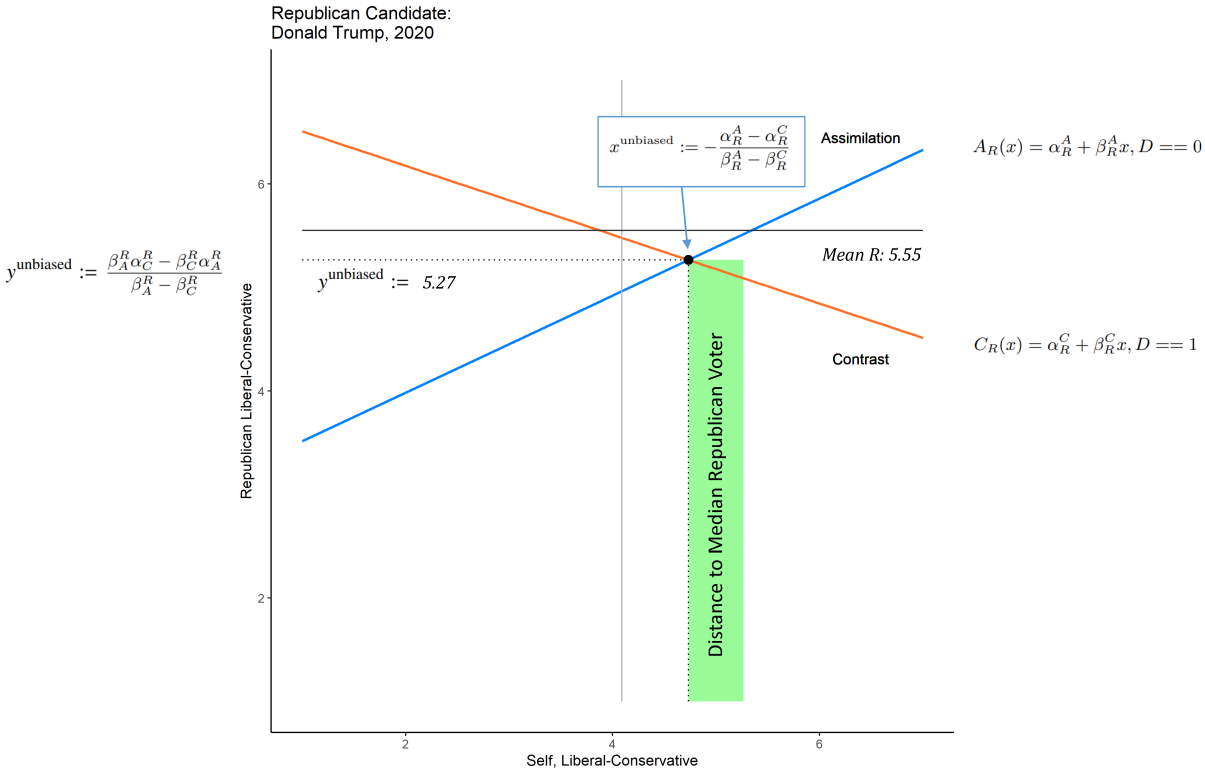
Therefore, a very Conservative respondent (a 7 in the horizontal scale), perceives Donald Trump as a conservative when voting for him (i.e. $3.05 + 0.469 * 7 = 6.333$ in Figure 1) and as a centrist when not voting for him (i.e. $6.83 - 0.332 * 7 = 4.05$ in Figure 1). Meanwhile, a very liberal respondent (a 1 in the horizontal axis), perceives Donald Trump as a moderate when voting for him ($3.05 + 0.469 * 1 = 3.519$ in Figure 1) and as conservative when not voting for him ($6.83 - 0.332 * 1 = 6.498$ in Figure 1).

A most interesting feature of the A-C lines is that there is a motivated downsian voter that will report the exact same location when supporting or opposing Donald Trump, $A_R(x) = C_R(x)$. For this voter, a change in allegiance to the "other" party (from the assimilated to the contrasted lines), will have no effect on the reported location of Donald Trump. This

[11] The only assumption in Figure 1 is that the relationship between the self-placement of the voters and the placement of the party is linear.

[12] Model results from the 2020 American National Election Study, with standard errors in parentheses, are: $A_R(x) = 3.05(0.097) + 0.469(0.017) * x$ and $C_R(x) = 6.83(0.074) - 0.332(0.023) * x$.

Fig. 2. Assimilation and Contrast, Donald Trump, NES 2020 Election



Note: Assimilation and Contrast (A-C) estimated using data from the American National Election Study (ANES, 2019). Lines describe the relationship between the self-reported conservatism of each respondent (x-axis) and the reported conservatism of Donald Trump jr. (y-axis). A-C lines from the survey data with model specification $A_R(x) = \alpha_R^A + \beta_R^A x$ when D==0 and $C_R(x) = \alpha_R^C + \beta_R^C x$ when D==1. Model results for Trump from the American National Election Study, with standard errors in parentheses, are: $A_R(x) = 3.05(0.097) + 0.469(0.017) * x$ and $C_R(x) = 6.83(0.074) - 0.332(0.023) * x$. Each voter observes Trump in either an assimilated or contrasted location. There exist an unbiased voter, $x^{\text{unbiased}} := -\frac{\alpha_R^A - \alpha_R^C}{\beta_R^A - \beta_R^C}$, who reports the same location for Trump irrespective of its membership into one of the A-C groups. All other voters, as they collect information that validates their preferences, observe Donald Trump in a position that is closer (assimilated) or further removed (contrasted) from another voter with similar self-reported conservatism that belongs to the "other" A-C group. The unbiased voter's perception of the party's position, which we call the party's *unbiased position*, is $y^{\text{unbiased}} := A(x^{\text{unbiased}}) = C(x^{\text{unbiased}}) = \frac{\beta_D^A \alpha_D^C - \beta_D^C \alpha_D^A}{\beta_D^A - \beta_D^C}$. Given that a majority of the conservative voters are *assimilated* and that a majority of the liberal voters are *contrasted*, the mean perceived ideological location of Donald Trump is shifted to the right of y^{unbiased}. The shaded area describes the distance between the $|x^{\text{unbiased}} - \bar{x}|$, with the self-reported conservatism of the unbiased voter located to the right of the mean supporter, $\bar{x}$.

unbiased voter is still a motivated reasoner, a rationalizing voter as described by Lodge and Taber (2013). However, the evidence she collects confirms an identical location when switching from one party to the other. The unbiased voter is located where $A_R(x)$ and $C_R(x)$ intersect:

$$x^{\text{unbiased}} := -\frac{\alpha_R^A - \alpha_R^C}{\beta_R^A - \beta_R^C}$$

The unbiased voter, as any other motivated voter, seeks out evidence that confirms her prior beliefs. However, different from the other voters, the evidence gathered by x^{unbiased} yields the same candidate (or party) location. For all other voters, confirmatory evidence places Donald Trump in a location that is closer (assimilated) or further removed (contrasted).

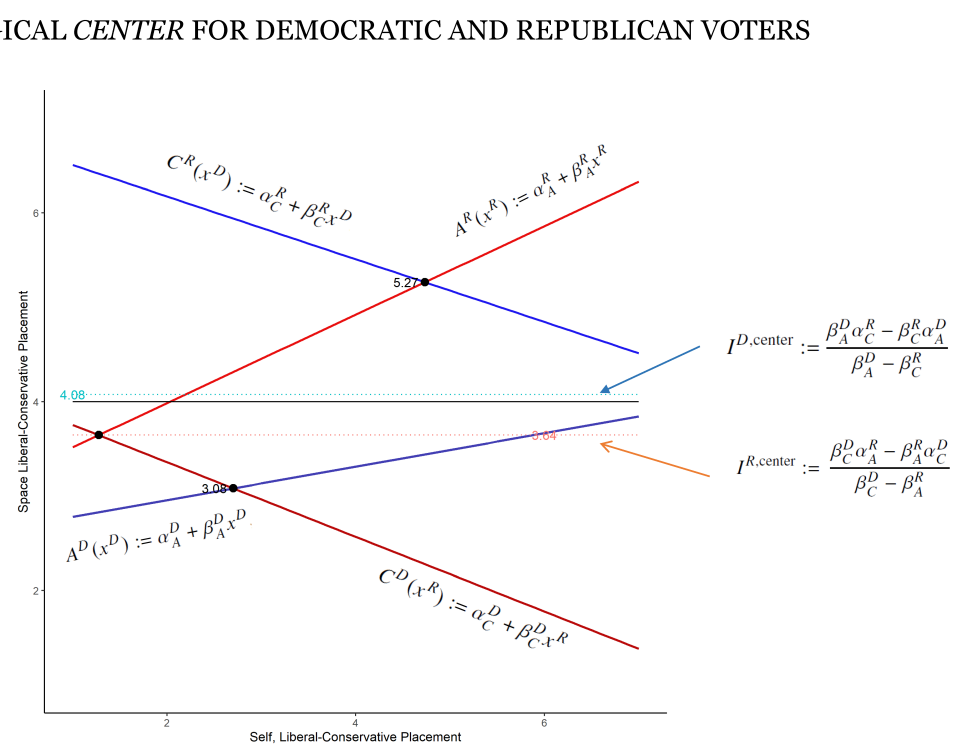
The location of the party as observed by x^{unbiased}, which we call the party's *unbiased position*, is given by:

$$\begin{aligned} y^{\text{unbiased}} &:= A(x^{\text{unbiased}}) = C(x^{\text{unbiased}}) \\ &= \alpha_R^A - \beta_R^A \frac{\alpha_R^A - \alpha_R^C}{\beta_R^A - \beta_R^C} = \frac{\beta_R^A \alpha_R^C - \beta_R^C \alpha_R^A}{\beta_R^A - \beta_R^C} \end{aligned}$$

For descriptive purposes, consider again the ANES data in Figure 2. Given that the majority of conservative voters are *assimilated* and that the majority of the liberal voters are *contrasted*, Figure 2 shows that the unweighted mean ideology of Donald Trump is more extreme, 5.55, than that retrieved from the unbiased party, y^{unbiased}, 5.27. In other words, because most Democrats perceive the Republican party further removed while most Republican perceive their preferred party closer to them, the unweighted mean is more extreme than its "unbiased" location.[13] Therefore, Figure 2 already already hints to a fundamental relationship between sorting

[13] This equation provides an alternative anchor to the latent model of Aldrich and McKelvey (1977).

Fig. 3. The center of the policy space observed by Democratic and Republican voters, ANES 2020 Presidential Election



Note: Assimilation and Contrast (A-C) for Joseph Biden and Donald Trump estimated using 2020 ANES data. In 2020, Democratic voters perceived the center of the policy space to be at 4.06 while Republican voters perceived the center to be at 3.69. A few interesting implications are readily observable from the equations: if negative campaigning against the Democratic Party shifts the constant α_C^R in the negative direction, so that the line $C^D(x^R)$ moves straight down, then the center of the political space observed by the Republicans will also move to the left, the unbiased Democratic voter($x^{D,\text{unbiased}}$)) moves left; the unbiased location of the Democratic party ($y^{D,\text{unbiased}}$)) moves left; while the center of the political space remains unchanged for the Democratic voters. Consequently, the distance between the center of the ideological space observed by the Democratic and Republican voters increases.

and polarization: it is not required that parties advertise more extreme ideological positions to observe parties as more extreme. Motivated downsian voters with higher assimilation and contrast estimates will generally observe an unweighted mean party ideology that is not only more extreme but also more polarized (i.e. more distant from the other party) than the one retrieved without perceptual bias. We will later show that the difference between the mean ideology and the unbiased ideology will increase when assimilation is larger and it will decrease when contrast is larger.

The position of the party reported by the unbiased voter will be useful in our description of the party's equilibrium in the final section of this article. We will update the location of each party by shifting y^{unbiased} to the left or right, with fixed assimilation and contrast slopes. The parties decision to move to the left or right, consequently, will be summarized by a change in the unbiased party position, $y^* - y^{\text{unbiased}}$.

In Figure 2, the green shaded area describes the distance between the position of the unbiased voter, where the A-C lines intersect, and the median party voter: $|x^{\text{unbiased}} - \bar{x_A}|$. In the case of Donald Trump, the self-reported conservatism of the unbiased voter is shifted to the left of the mean supporter, $\bar{x}_A$. We will come back to the difference between the unbiased voter and the median party voter after we describe the properties of the unbiased policy space: the "center".

The moving "center" and the shape of the ideology space. In the previous section we defined the "unbiased voter" as one that perceives a party in the same location when gathering confirmatory evidence. The "unbiased" motivated voter who abandons the Republican party to embrace the Democratic party will still observe the Republican party in the exact same location. It may still surprise readers to note that this same voter will see the Democratic party at a new location. That is, the "unbiased" voter who observes the Republican party as unchanged if she switches from "supporting" to "opposing" it, would still observe a change in the location of the Democratic party when she becomes a "supporter" (the exact same change).

This intuition is particularly relevant and deserves to be looked at with greater care. Unless the "unbiased" voter that observes the Republican party is the exact same "unbiased" voter that observes the Democratic party, a change in allegiance from supporting Republicans to supporting Democrats will only appear to affect the perceived location of the Democratic party while holding the other party's location constant.

For the "unbiased" voter that shifts from a Republican supporter to a Democratic supporter, only the Democratic party will appear closer to her in the ideological space.

This raises a critical question: When will both parties be perceived as identical for this "unbiased voter"? We may proceed as we did in the previous section and consider the location of an "unbiased" voter in the location of the ideological space were the two parties are identical (i.e. the "center" of the ideological space). For supporters of each party, $[x^D, x^R]$, this "center" of the ideological space will be the location where the parties' positions are perceived as (ideologically) identical: $A^D(x^D) = C^R(x^D)$ and $A^R(x^R) = C^D(x^R)$. It becomes immediately apparent that, as in the earlier example, there is a different center for the Democratic voter, x^D, and for the Republican voter, x^R. There is a Democratic voter who observes the "center" of the ideological space at:

$$x^{D,\text{center}} := -\frac{\alpha_C^R - \alpha_A^D}{\beta_C^R - \beta_A^D}.$$

Similarly, there is a Republican voter who observes the "center" of the ideological space at:

$$x^{R,\text{center}} := -\frac{\alpha_A^R - \alpha_C^D}{\beta_A^R - \beta_C^D}.$$

The "unbiased" ideological space position of these two voters allow us to derive the "center" of the ideological space. For the Democratic voters this is:

$$I^{D,\text{center}} := \alpha_A^D - \beta_A^D \frac{\alpha_C^R - \alpha_A^D}{\beta_C^R - \beta_A^D} = \frac{\beta_A^D \alpha_C^R - \beta_C^R \alpha_A^D}{\beta_A^D - \beta_C^R}.$$

In our ANES 2020 example, in Figure 3, this location is described by a dotted blue line located at $I^{R,\text{center}} = 4.06$. Similarly, the "center" of the ideological space for the Republican voters is:

$$I^{R,\text{center}} := \alpha_A^R - \beta_A^R \frac{\alpha_A^R - \alpha_C^D}{\beta_A^R - \beta_C^D} = \frac{\beta_C^D \alpha_A^R - \beta_A^R \alpha_C^D}{\beta_C^D - \beta_A^R}.$$

In our ANES 2020 example, again in Figure 3, this location is described by a dotted red line located at $I^{R,\text{center}} = 3.64$. At this point it should be readily clear to the readers that when parties move on the ideological space, the perceived "center" changes for the Democratic and Republican voters. The perceived "center" of the ideological space will differ for Democratic and Republican voters and, as important, the ideological space will also differ within the parties. We will later show that, if $I^{R,\text{center}}$ is below the median, the voters perceive this party as too far right. Meanwhile, if $I^{R,\text{center}}$ is above the median, this party is perceived as too far left. For most of the elections between 1972 through 2020, both the Democratic and Republican parties were perceived by their voters as "shifted to the left". The 2020 is an uncommon election and one that we will further analyze later in section 3.

Our formal treatment provides numerical estimates of the changes in the perceived "center" of the ideological space. More important, however, the natural extension of this intuition is that voters who switch allegiances from one party to the other will perceive not only that parties move but also that the position of ***all other voters move***, as the space stretches or contracts. This is similar to the description of Aldrich and McKelvey (1977), although in their article they assume that the space has a single center and each party has a "true" location.

Except for the specific case where the median voter is also the $I^{[R,D]}$, parties cannot be observed as converging to the same position. In other words, if Republicans see both parties as ideologically identical (i.e. the "center"), Democratic voters will perceive the parties as not identical (i.e. not at the "center"). Consequently, there is no convergence to the median voter that would be simultaneously observed by all voters. This finding is independent of whether parties move centripetally or centrifugally and allows us to discriminate the extent to which voters perceive their party as "too liberal" or "too conservative".[14]

In all, for any voter x with a fixed self-reported ideological location, perceptual bias (assimilation and contrast) will alter both the perceived location of parties as well as the perceived location of all other voters, including the median voter. A voter with a self-reported fixed location x, therefore, observes a median voter that is both different from the "center" of the ideological space and that changes its location when parties move.

2. On the effect of non-ideological shocks: when *assimilation* and *contrast* change

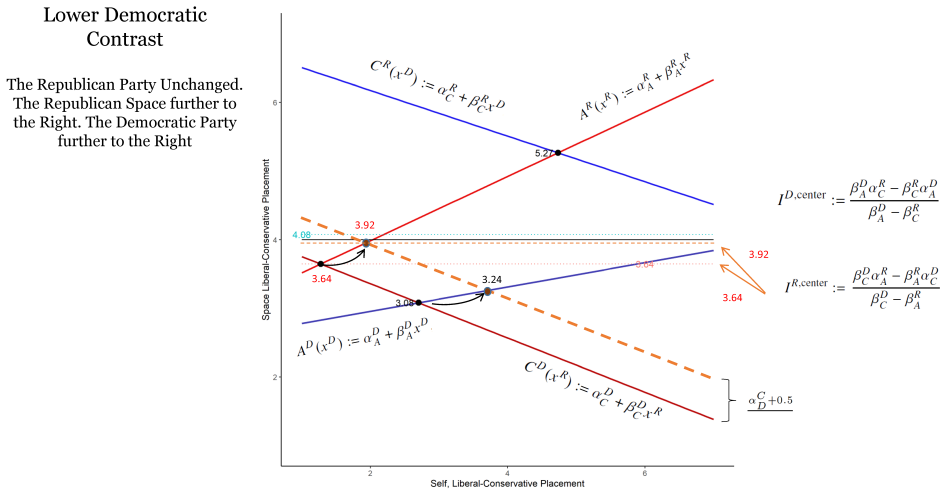
Once we define $A^D(x^D)$, $A^R(x^R)$, $C^D(x^R)$, and $C^R(x^D)$, and solve for the two unbiased party locations and the two "centers" of the policy space, we may derive a number of interesting implications. Before we describe the comparative statics and the equilibrium location, let us consider two examples that provide an intuition of how changes in assimilation and contrast alter the perceived location of the parties and "center" of the space.

Lowering contrast and moving towards the "other" party. Consider the positive policy shock in Figure 4, which reduces contrast for Joe Biden among Republicans by a constant value, $\alpha_D^{C*} = \alpha_D^C + 0.5$. The policy shock moves the Democratic contrast line upward, $C^D(x^R)$. As the line moves up, the unbiased location of the democratic party also changes to $y^{\text{unbiased}} = 3.24$, from the previous value of 3.08. Therefore, the change in the line results in a more moderate location for the Democrats. However, the "center" of the ideological space among Republicans changes to $I^R = 3.92$ from its previous location of 3.64. The interpretation of this change is that, before contrast was updated, the location at which parties would be identical was 3.64 (i.e. the space was on the left and the party was too far right). After contrast is lowered, however, the space compresses and the Republican party is now at 3.92, no longer "too far right" but rather almost at the median of the space.

We can see that, using ANES data from 2020, a constant positive change in the contrast line for Biden moved the center of the ideological space, $3.92 - 3.64 = .28$, faster than the perceived moderate shift in the location of the Democratic party, $3.24 - 3.08 = .16$. For Republicans, as the Democratic party moved in their direction the "center" of the space ap-

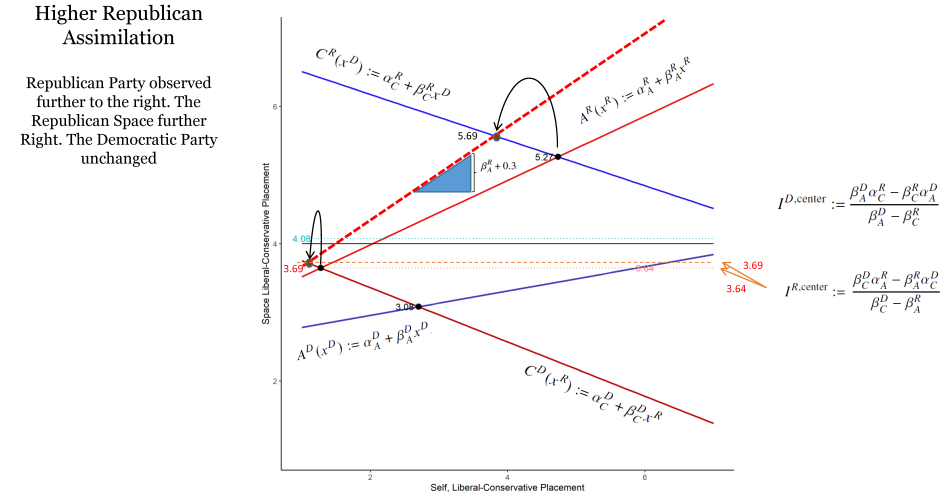
[14] For a special case for which the "center" of the ideological space is the same irrespective of the party voter see the appendix.

Fig. 4. A decrease in Contrast among opponents of the Democratic Party



Note: A positive shock on the contrast line moves the Democratic Party to the right. However, the ideological "center" observed by the Republicans shows a wider jump. The Democrats are only observed as more "moderate" (more conservative) by the Democratic voters. Meanwhile, for Republican voters the Democrats are still "too slow to react" and lag the change to the right by "all voters".

Fig. 5. An increase in Assimilation among supporters of the Republican Party



Note: An increase in assimilation moves the Republican Party to the right. There is almost no change in the ideological "center" observed by voters of both parties. The Democrats are only observed as more conservative (centripetal change) by the Democratic voters. Meanwhile, for Republican voters the Democrats are still "too slow to react" and lag the change to the right by "all voters". Assimilation and Contrast (A-C) estimated using data from the American National Election Study (ANES, 2019).

proximates. So much so, that their party is no longer "too far left".

Another interpretation is that the positive policy shock reduced the overall distance between the Republican party and all voters at a faster pace than the change in the Democratic party. A Republican voter would say: "Everyone is now closer to us except, as usual, the Democratic Party". For a Democratic voter, however, the center of the policy space has not changed. Only the unbiased position of the party moved in a conservative direction. After the policy shock, the

political space compressed but the perceived distance between the parties and the center of the ideological did not became smaller. In fact, Democratic voters on the very left of the political spectrum will perceive that the party moved away from them while Republican voters will not perceive that the Democratic party is any closer. Rather, Republican voters will perceived that their party is now a more moderate party.

When Assimilation Increases. Defining $A^D(x^D)$, $A^R(x^R)$, $C^D(x^R)$, and $C^R(x^D)$, and then solving for the two unbiased party locations and the two "centers" of the policy space, also allows us also to explore the effect of changes in the assimilation and contrast slopes.

Let us exemplify analyzing a positive change in assimilation for Trump, which results in voters perceiving Trump ideologically closer. Me may add a constant of 0.3 to the assimilation slope, $\beta_R^{A*} = \beta_R^A + 0.3$. The shock makes the assimilation line steeper, moving the unbiased location of the Republican party to $y^{\text{unbiased}} = 5.69$, from the previous value of 5.27. Therefore, the change in the line results in Republican candidate that is perceived as more conservative. In this case, the "center" of the ideological space among Republicans bearly moves, reaching $I^R = 3.69$ from its previous location of 3.64. Using the 2020 ANES data, the example shows that a positive change in the assimilation slope moves the Republican party to the right faster than the change in the perceived "center" of the ideological space. For both Republicans and Democrats, the party is to the right.

Assimilation and Contrast: General View

Figure 6 describes the dynamics of the model in more general terms. We demean our data allowing the scale of the x-axis and y-axis to be centered at 0. Therefore, a change in the constant will only move the lines up or down. Similarly, if the unbiased voter of both parties is located at 0, the slope will only describe rotations of the ideological space.

Placed in this form, the overall intuition comes across clearly: all changes in assimilation and contrast slopes, $\beta_{A,C}^{D,R}$, will only rotate the lines and alter the perceived "center" of the policy space. If β_A^D increases, the unbiased democratic voter I_x^D will observe the "center" move in the direction of the Republican space, the 'out-group'. That is, the Democratic voter will expect the Democratic party to be identical to the Republican party in a location that is further away from them and closer to the Republican party. This is equivalent to perceiving that the party is "too far left" and expecting that it will adjust by moving to the right. Therefore, higher assimilation moves the "center" observed by the Democrats in the direction of the Republicans and increases the distance from the center to the party.

Similarly, an increase in the assimilation slope of the Republican party, β_A^R, will move the "center" observed by the Republican voters towards the Democratic voter, with the party being perceived as "too far right". In all, lower levels of assimilation increase the pressure to move towards 'in-group' voters while higher levels of assimilation increase the pressure to move towards 'out-group' voters.

If the contrast slope of the Democratic party increases, β_C^D, the unbiased Republican voter I_x^R will observe the "center" moving in the Republican direction and their party as being "not conservative enough". That is, for Republican voters a more contrasted Democratic party will not only result in perceiving Democratic voters as too leftist but also in perceiving a Republican party that is not rightist enough ("not conservative enough"). Similarly, a less contrasted Democratic party will result in Democrats being perceived as less extreme and Republicans as being "conservative enough", at least in relative terms.

The opposite is true for the a contrasted Republican party: a larger more negative slope β_A^R will move the "center" of the Democratic space towards the in-group, increasing the perception of a Republican party that is too far right and a Democratic party that is "not progressive enough". A less contrasted Republican party, on the other hand, will be perceived as more centrist while the Democratic party is perceived as "progressive enough".

Figure 6 shows that an actual change in the policy space, a left or right shift in the intercepts $\alpha_{A,C}^{D,R}$, has very different consequences. When $\alpha_{A,C}^{D,R}$ increases centripetally, towards the out-group party, both the unbiased location of the party as well as the ideological space moves towards the out-group. The rate of the adjustment, however, depends on the relative values of the slopes. Because in Figure 6 all slopes are equal in magnitude (with contrast being negative and assimilation positive), a change in $\alpha_{A,C}^{D,R}$ moves both the parties and the space at constant rates and the relative distances to the center remain unchanged. This critical results shows that if the Democratic or Republican party move to the left or right, so that only the intercepts change, most voters will perceive that the ideological space has shifted and that the distances from the parties to the center remain unchanged. To be perfectly clear, there is no centripetal movement in the space that will change the relative distances to the "center" unless the assimilation and contrast slopes of the parties are of different magnitudes.

However, Figure 6 readily show that shocks that move $\alpha_A^{D,R}$ or $\alpha_C^{D,R}$ independent from each other, will only alter the ideological space observed by one of the two groups of voters. Therefore, while a party that "announces" moderation will be met with an equivalent adjustment of the ideological "center", a shock that affects only the assimilation or contrast constants will produce actual divergence between the parties.

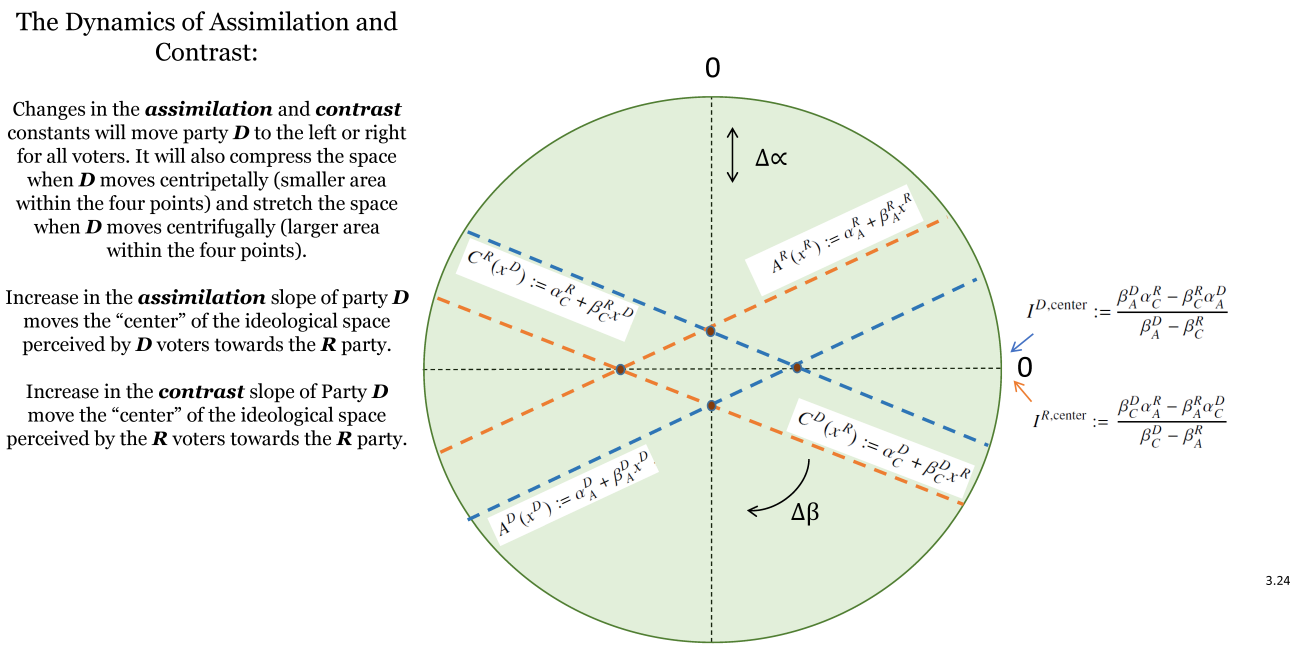
3. Analyzing ANES results: 1972-2020

Now that we described in detail how changes in assimilation and contrast alter the perceived location of the parties and the center of the ideological space, we are ready to explore the ANES survey data from 1972 through 2020. For each of the 13 elections from 1972 through 2020, we estimate the four assimilation and contrast equations:

$$
\begin{aligned}
A^R(x^R) &:= \alpha_A^R + \beta_A^R x^R, \\
A^D(x^D) &:= \alpha_A^D + \beta_A^D x^D, \\
C^R(x^D) &:= \alpha_C^R + \beta_C^R x^D, \\
C^D(x^R) &:= \alpha_C^D + \beta_C^D x^R,
\end{aligned}
$$

We fit a multilevel model with random intercepts and slopes by election/year. Mean parameter estimates are presented in Table 1 while the estimates with their confidence intervals are in Figure 7.

Fig. 6. The Model of Assimilation and Contrast



Note: General dynamics of assimilation and contrast. Axes demeaned, allowing the scale of the x-axis and y-axis to be centered at 0. Therefore, a change in the constant will only move the lines up or down. Similarly, if the unbiased voter of both parties is located at 0, the slope will only describe rotations of the ideological space.

Even before we conduct any further analysis, it is immediately apparent that for most of the last fifty years Republican voters have been more contrasted than assimilated and that the opposite is true among Democrats. By the rules described in the general model we expect that the center of the political space will be in both cases shifted to the right, with Democrats perceiving that the party is "too progressive" and the Republicans perceiving that their party is "not conservative enough". Two exceptions are the re-election of Ronald Reagan in 1984 and the loss of Donald Trump against Joseph Biden in 2020. In both of those elections, we see sharp increases in assimilation and, in the case of Trump, a return to Clinton era levels of contrast. In other words, while between 2004 and 2016 Republican voters perceive that their party was "not conservative enough", that was no longer the case in the failed re-election of Trump in 2020.

Figure 8 summarizes the location of the parties as observed by the unbiased voters as well as the shift in the ideological space by the $I_x^{D,R}$ voters. As it is possible to observe, the Democratic party made a significant rightward shift in failed re-election of Jimmy Carter in 1980, which was unable to compensate for the Democratic voters perception that their party was "too far left". Perceptions of the Democratic party as being too liberal persisted until the failed election of Al Gore. Since 2000, the Democratic Party has been shifting to the left while pressure to move rightward decline. In 2016, for the first time since 1972, the Democrats perceived that their party was not progressive enough while the Republicans perceived that their party was not conservative enough. Indeed, the election of Donald Trump may be considered a particularly polarized one not only because the distance between the parties was large but also because both Republicans and Democrats perceived that their parties were too moderate.

While the estimates in Figure 7 simply summarize the model estimates for each election and candidate, Figure 8 adds significant information which depends critically on accepting some assumptions of the model: (1) the linearity assumption in the estimation of assimilation and contrast;[15] (2) the assumption that at the point where the assimilation and contrast party lines intersects there is a motivated downsian voter that observes the party in an identical position; and (3) our interpretation of an unbiased voter that observes the center of the policy space where the assimilation and contrast lines of each group intersect.

4. An experimental assessment of assimilation and contrast on social media exposure

It is our believe that there is valuable and novel information in our re-estimation of the presidential election series of the American National Election Studies (ANES, 2019). However, there are more interesting applications of the proposed approach, particularly when it is used to gain further insights from existing (or future) experimental data.

We now reexamine a recent study by Banks et al. (2021) that measures the effect of social media exposure on polarization. The experiment asks the respondents to place themselves,

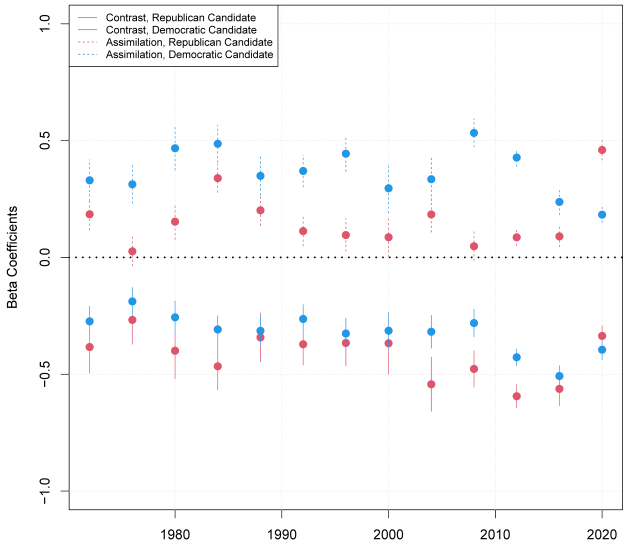
[15] See a variation using a quadratic specification in the Appendix

Table 1. Assimilation and Contrast Parameters, ANES 1972-2020

Year	α_C^R	β_C^R	α_A^R	β_A^R	α_C^D	β_C^D	α_A^D	β_A^D
1972	0.974	-0.384	0.753	0.184	-1.493	-0.274	-1.063	0.330
1976	1.095	-0.268	0.824	0.026	-0.736	-0.189	-0.509	0.313
1980	1.235	-0.400	1.005	0.153	-0.245	-0.257	0.133	0.467
1984	1.098	-0.466	0.868	0.339	-0.607	-0.309	-0.208	0.486
1988	1.258	-0.343	1.017	0.201	-0.859	-0.314	-0.341	0.349
1992	1.257	-0.372	1.070	0.112	-0.942	-0.264	-0.425	0.370
1996	1.244	-0.366	1.195	0.095	-1.078	-0.326	-0.345	0.444
2000	1.224	-0.368	1.008	0.086	-0.916	-0.314	-0.559	0.296
2004	1.243	-0.543	1.188	0.184	-1.131	-0.318	-0.576	0.335
2008	0.838	-0.477	1.002	0.047	-1.003	-0.281	-0.345	0.533
2012	0.985	-0.594	1.058	0.086	-1.424	-0.428	-0.571	0.427
2016	0.780	-0.563	0.856	0.090	-1.329	-0.508	-0.722	0.237
2020	1.499	-0.336	0.936	0.460	-1.431	-0.395	-0.685	0.183

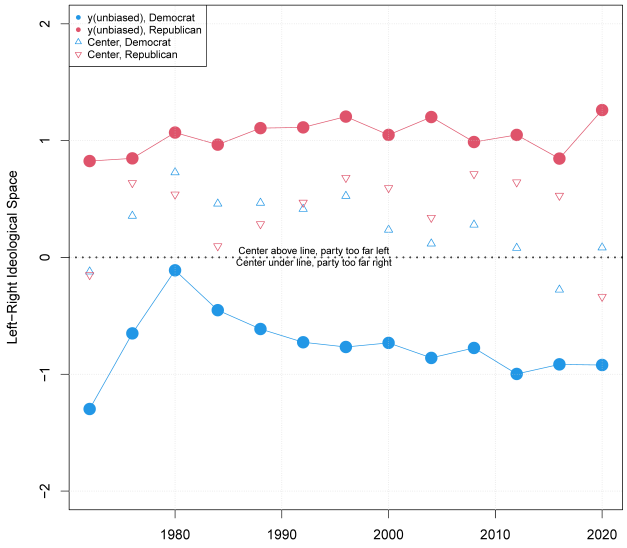
Note: Mixed model estimation (LMER). SE omitted from the table to simplify the interpretation of the results. See Appendix.

Fig. 7. Assimilation and Contrast, Parameter Estimates, ANES 1972-2020



Note: Estimates of assimilation and contrast by party and year. American National Election Study.

Fig. 8. Location of the $y^{k,\text{unbiased}}$ and $I_x^{D,R}$, ANES 1972-2020



Note: Red solid circles describe the location of the $y^{R,\text{unbiased}}$ and blue solid circles describe $y^{D,\text{unbiased}}$.

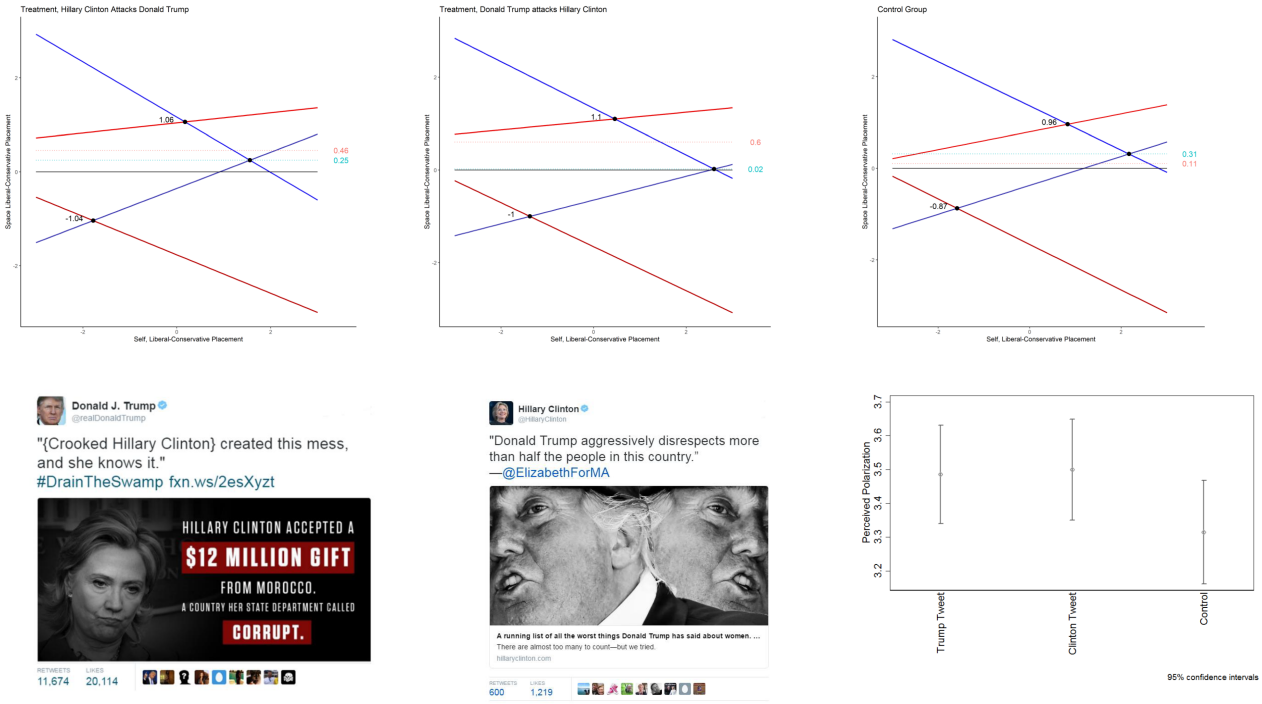
Donald Trump, and Hillary Clinton in the ideological space after the 2016 election. The researchers used the same 7 point ideological scale question of the ANES, thereby adding an experimental treatment while preserving the exact same scales we used in the previous section.

The goal of the study was to measure perceived polarization after 2/3 of survey respondents were treated with negative attacks from the out-group candidate. A total of 1/3 of the respondents were treated to a negative tweet by Donald Trump, who accuses Hillary Clinton of corruption and embezzlement. Another 1/3 of the respondents was treated to a negative tweet by Hillary Clinton, who accuses Donald Trump of being a sexual predator. Finally, the remaining 1/3 of respondents was kept as a control group.

In their article, the authors compare the unweighted mean location reported by the Democratic and Republican voters, showing that among those exposed to the tweets the perceived distance increased by a statistically significant 0.2 points, from approximately 3.3 for the control group to 3.5 for those treated with either tweet.

In this section, we reexamine the original data, estimating assimilation and contrast lines for each party and treatment of their study. The three different sets of parameters are reported in Tables 3, 4, and 5, estimating the models (1) $C^D(x^R)$, (2) $A^D(x^D)$, (3) $C^R(x^D)$, and (4) $A^R(x^R)$. Each of these models estimates the β and α parameters, allowing us to compute the positions of the unbiased voters, the unbiased party locations, and the unbiased ideological space.

Fig. 9. The Assimilation and Contrast Experiment, Banks et al. 2021



Note: Experimental data from Banks et al. (2021). One third of the respondents were treated with Donald Trump's tweet, one third were treated with Hillary Clinton's tweet, and one third kept as control. The treated respondents reported higher perceived polarization. Upper plots describe the assimilation and contrast lines for each of the groups.

Table 2. Party and Space Locations

Year	yD	yR	ySD	ySR
1972	-1.298	0.825	-0.122	-0.150
1976	-0.651	0.847	0.355	0.637
1980	-0.111	1.069	0.727	0.539
1984	-0.452	0.965	0.459	0.096
1988	-0.613	1.107	0.466	0.285
1992	-0.727	1.113	0.414	0.469
1996	-0.767	1.205	0.525	0.681
2000	-0.732	1.049	0.236	0.594
2004	-0.861	1.202	0.117	0.339
2008	-0.776	0.988	0.279	0.713
2012	-0.997	1.048	0.080	0.643
2016	-0.916	0.846	-0.277	0.528
2020	-0.921	1.261	0.084	-0.337

Let us describe some of the most important results in the three models. First, we can see that treated respondents lower their assimilation and increase contrast with Trump. The increase in contrast results in Democrats perceiving that the Democratic party is "not progressive enough" while the decrease in assimilation results in Republicans perceiving that the Republican party is "not conservative enough". Both changes will move the ideological space towards in-group supporters and increase perceived polarization. The results is that the "center" of the ideological space for Republicans moved from $I_x^R = 0.11$ in the control group to $I_x^R = 0.46$ (negative Trump Tweet), and $I_x^R = 0.60$ (negative Clinton Tweet). By contrast, the reported location of the unbiased party only increased modestly, from 0.96 to 1.06 and 1.1 respectively. In all, the main effect of the experiment among Republican voters was to reduce assimilation for Trump and to displace the perceived "center" to the right.

Among Democrats, we see that the negative tweets lowered contrast and increased assimilation. The decline in contrast and increase in assimilation are statistically significant for the first treatment (Trump's attack on Clinton). The effect is a modest left shift for the Democratic party and a similar shift in the ideological space among Democrats. While the overall difference between the parties is similar in both treatments (i.e. $1.06 + 1.04 = 2.1$ after Trump's Tweet and $1.1 + 1 = 2.1$ after Hillary Clinton's tweet), the difference in the perceived "center" is significantly larger for the Hillary Clinton's treatment (i.e. $0.6 - .02 = .58$ after Clinton's Tweet compared to $0.46 - 0.25 = 0.21$ after Donald Trump's tweet).

Table 3. Donald Trump Attacks Hillary Clinton

	Contrast Clinton $C^D(x^R)$	Assimilation Clinton $A^D(x^D)$	Contrast Trump $C^R(x^D)$	Assimilation Trump $A^R(x^R)$
self-β	-0.409^{***}	0.385^{***}	-0.588^{***}	0.107
	(0.075)	(0.069)	(0.090)	(0.079)
Constant-α	-1.766^{***}	-0.351^{***}	1.165^{***}	1.043^{***}
	(0.130)	(0.120)	(0.154)	(0.139)
N	151	161	157	150
Adjusted R^2	0.159	0.159	0.213	0.006
F Statistic	29.463^{***} (df = 1; 149)	31.167^{***} (df = 1; 159)	43.100^{***} (df = 1; 155)	1.831 (df = 1; 148)

Statistical Significance: ***<.01. **<.05. *<.1

Table 4. Hillary Clinton attacks Donald Trump

	Contrast Clinton $C^D(x^R)$	Assimilation Clinton $A^D(x^D)$	Contrast Trump $C^R(x^D)$	Assimilation Trump $A^R(x^R)$
self-β	-0.475^{***}	0.257^{***}	-0.504^{***}	0.095
	(0.064)	(0.065)	(0.096)	(0.073)
Constant-α	-1.655^{***}	-0.648^{***}	1.330^{***}	1.056^{***}
	(0.115)	(0.113)	(0.166)	(0.131)
N	150	159	157	150
Adjusted R^2	0.269	0.083	0.147	0.005
F Statistic	55.877^{***} (df = 1; 148)	15.391^{***} (df = 1; 157)	27.816^{***} (df = 1; 155)	1.721 (df = 1; 148)

Statistical Significance: ***<.01. **<.05. *<.1

Table 5. Control Group

	Contrast Clinton, $C^D(x^R)$	Assimilation Clinton $A^D(x^D)$	Contrast Trump $C^R(x^D)$	Assimilation Trump $A^R(x^R)$
self-β	-0.497^{***}	0.316^{***}	-0.483^{***}	0.195^{**}
	(0.070)	(0.078)	(0.096)	(0.075)
Constant-α	-1.662^{***}	-0.371^{***}	1.363^{***}	0.800^{***}
	(0.124)	(0.136)	(0.166)	(0.132)
N	164	142	140	163
Adjusted R^2	0.233	0.098	0.149	0.034
F Statistic	50.554^{***} (df = 1; 162)	16.286^{***} (df = 1; 140)	25.373^{***} (df = 1; 138)	6.769^{**} (df = 1; 161)

Statistical Significance: ***<.01. **<.05. *<.1

In all, we see that the treatments increase the extent to which Republican voters perceive their party as "not conservative enough" and reduce the perception among Democratic voters that their party is "too far left". However, there is significant information that was previously unavailable, as we see the different moving parts of the treatment. Most important, we now can readily see that the while the "unbiased" distance between the parties increased by similar amounts after treatments, Trump's attacks had a more dramatic effect in increasing the distance in the ideological space. Everything else equal, Donald Trump's negative attacks on Hillary not only increase the perceive distance between the parties (polarization) but also how far the parties should move to meet the in-group demands.

5. Concluding Remarks

Fourty years ago, Aldrich and McKelvey (1977) made a call to model perceptual bias explicitly and to incorporate such biases into existing downsian models of the vote. Their scaling technique sought to find a common metric, an anchor, that would reduce extra variation in the data and place all voters in a common space. Twenty five years later, Merrill, Adams, and Grofman (2001) and Adams, Merrill, and Grofman (2005) made explicit their concern that attenuation biases in equilibrium models of voting could be the result of attenuation biases.

This article hopes to further advance this agenda and to model the attitudinal consequences of *assimilation* and *contrast* when parties move in the space and when voters change their party preferences. We extract more information from survey data by asking two critical questions that can be tested with existing data: (1) how would the perceived positions of the parties change if one voter switches party preferences at the location were the assimilation and contrast lines intersect for one of the parties? Second, (2) what is the location in the space at which the two parties would be perceived as indistinguishable from each other by a motivated reasoner?

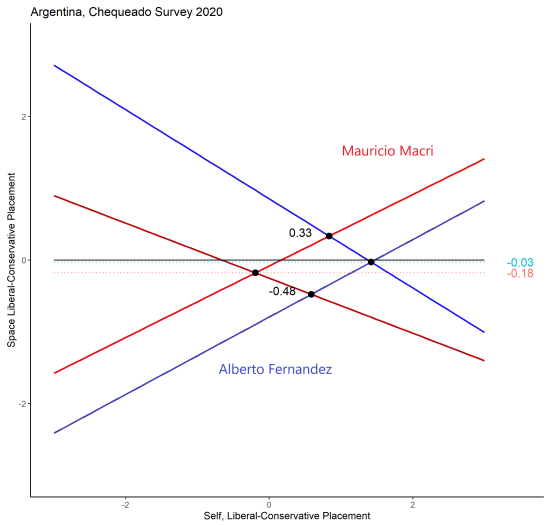
In answering these two questions, we show that changes in the ideological location of the parties will also alter the relative distances among voters as well as the perceived liberal or conservative leaning of the political system. We are able to derive unbiased positions for the parties (i.e. anchors) but also show the anchors for each of the parties will not converge to a "center" (i.g. a fixed location) that is observed by in-group and out-group members.

Results from our analysis yield estimates of "how conservative is politics in the United States" that differ for Democratic and Republican observers. Further, our reanalysis of experimental survey data illustrates that increased polarization can be further decomposed in a distance between the parties and a distance between the locations where both parties would be perceived as identical.

As we move to extend our study to other countries, some interesting extensions of the theory will be required. Estimates in other countries, such as the Argentine example in Figure 10, should allow for better estimation of the determinants of the changes in the ideological space.

In multiparty systems, however, the location of the unbiased space will have multiple measurements by party, increasing variance not only across parties but also within parties. Future research will benefit from a better understanding of the different locations at which parties will be perceived as identical when there are more than two candidates.

Fig. 10. Argentina 2020 Social Media Survey, reported location of conservative candidate Mauricio Macri and peronist candidate Alberto Fernandez



Note: Estimates of unbiased voter and unbiased space from the 2020 Factchecking Survey, Chequeado, Argentina, March 2020. Table with estimates in the Appendix

ACKNOWLEDGMENTS.

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Assimilation and contrast – Appendix

By Ernesto Calvo, Kiyoung Chang, *and* Juan Dodyk

I. Basic Notation and Concepts

A. *Assimilation, contrast, unbiased voter, and unbiased party location*

There is a party. People can be supporters or not. A supporter with ideal point x (in a one-dimensional policy space) perceives the position of the party as

$$A(x) := \alpha_A + \beta_A x,$$

with $\beta_A > 0$ (*assimilation*). If the individual does not support the party, her perception of the party's position is

$$C(x) := \alpha_C + \beta_C x,$$

with $\beta_C < 0$ (*contrast*).

A voter is *unbiased* if her perception is independent of her support to the party, i.e., if $A(x) = C(x)$. Solving for x, the position of the unbiased voter is:

$$x^{\text{unbiased}} := -\frac{\alpha_A - \alpha_C}{\beta_A - \beta_C}.$$

The unbiased voter's perception of the party's position, which we call the party's *unbiased position*, is:

$$y^{\text{unbiased}} := A(x^{\text{unbiased}}) = \alpha_A - \beta_A \frac{\alpha_A - \alpha_C}{\beta_A - \beta_C} = \frac{\beta_A \alpha_C - \beta_C \alpha_A}{\beta_A - \beta_C}.$$

B. *Ideological space*

Each voter observes two parties, which for presentation purposes we define as Democratic and Republican. Supporters of each party observe the assimilated position:

$$A^R(x^R) := \alpha_A^R + \beta_A^R x^R,$$
$$A^D(x^D) := \alpha_A^D + \beta_A^D x^D,$$

with $\beta_A^{R,D} > 0$.

If the individual does not support a party, her perception of the party's position is:

$$C^R(x^D) := \alpha_C^R + \beta_C^R x^D,$$
$$C^D(x^R) := \alpha_C^D + \beta_C^D x^R,$$

with $\beta_C^{R,D} < 0$.

Supporters in each party, $[x^D, x^R]$, observe the "center" of the ideological space at the location where parties' positions are perceived as (ideologically) identical: $A^D(x^D) = C^R(x^D)$ and $A^R(x^R) = C^D(x^R)$. Solving for x^D and x^R, the Democratic voter that observes the "center" of the ideological space is:

$$x^{D,\text{center}} := -\frac{\alpha_C^R - \alpha_A^D}{\beta_C^R - \beta_A^D}.$$

Meanwhile, the Republican voter that observes the "center" of the ideological space is:

$$x^{R,\text{center}} := -\frac{\alpha_A^R - \alpha_C^D}{\beta_A^R - \beta_C^D}.$$

From the position of these two voters we may derive the "center" of the ideological space for the Democratic voters as:

$$I^{D,\text{center}} := \alpha_A^D - \beta_A^D \frac{\alpha_C^R - \alpha_A^D}{\beta_C^R - \beta_A^D} = \frac{\beta_A^D \alpha_C^R - \beta_C^R \alpha_A^D}{\beta_A^D - \beta_C^R}.$$

Similarly, the "center" of the ideological space for the Republican voters is:

$$I^{R,\text{center}} := \alpha_A^R - \beta_A^R \frac{\alpha_A^R - \alpha_C^D}{\beta_A^R - \beta_C^D} = \frac{\beta_C^D \alpha_A^R - \beta_A^R \alpha_C^D}{\beta_C^D - \beta_A^R}.$$

A most interesting result is that as parties move in the ideological space, the perceived "center" changes for the Democratic and Republican voters. Except for the specific case where the median voter is also the $I^{[R,D]}$, parties cannot be observed as converging to this position. In other words, if Republicans see both parties as ideologically identical (i.e. the "center"), Democratic voters will perceive the parties as not identical (i.e. not at the "center").

Therefore, there is no convergence to the median voter that would be simultaneously observed by all voters. This finding is independent of whether parties move centripetally or centrifugally.

There is a special case for which the "center" of the ideological space is the same irrespective of the party voter. This extreme outcome will be observed when $I^R = I^D$, the location at which all voters observe the same "center". The global ideological "center" is given by the equation:

$$I^{G,\text{center}} := \frac{\beta_A^D * (\beta_C^D * (-\alpha_A^R + \alpha_C^R) + \beta_A^R * (\alpha_C^D - \alpha_C^R)) + \beta_C^R * (-\alpha_C^D * \beta_A^R + \alpha_A^R * \beta_C^D)}{(\beta_C^R * (-\beta_A^R + \beta_C^D))}$$

However, for most combinations of assimilation and contrast parameters, when either party moves in the ideological space voters will also perceive a change in the ideological "center" of the space. An alternative interpretation of these formulas is that, for any voter x with a fixed self-reported ideological location, perceptual bias (assimilation and contrast) will alter both the perceived location of parties as well as the perceived location of all other voters, including the median voter. A voter with a self-reported fixed location x, therefore, will observes a median voter that is both different from "center" of the ideological space and that changes its location when parties move.

II. Equilibrium Algorithm

We have a set of voters $\mathcal{I} = \{1, \ldots, I\}$ and a set of parties $\mathcal{K} = \{1, \ldots, K\}$. Voter $i \in \mathcal{I}$ has ideology $x_i \in \mathbb{R}$ and voted for $v_i^0 \in \mathcal{K}$ in the past election. Party $k \in \mathcal{K}$ has assimilation and contrast parameters $\alpha_k^A, \alpha_k^C, \beta_k^A, \beta_k^C \in \mathbb{R}$ (with $\beta_k^A > 0$, $\beta_k^C < 0$), and chooses a shift $t_k \in \mathbb{R}$ in her position. A voter i receives utility $U_{ik} = u_{ik} + \epsilon_{ik}$ from voting for k, where

$$u_{ik} = \begin{cases} -A(\alpha_k^A + t_k + \beta_k^A x_i - x_i)^2 & \text{if } i \text{ supports } k, \\ -A(\alpha_k^C + t_k + \beta_k^C x_i - x_i)^2 & \text{otherwise,} \end{cases}$$

and ϵ_{ik} are iid and follow a Type 1 extreme value distribution. Let $v_i \in \mathcal{K}$ be the party that i votes for.

We look for an equilibrium in which (1) the parties choose t_k to maximize the expected number of votes, and (2) voters support the party they vote for.

Given who each voter votes for, the expected number of voters for k is $V_k = \sum_{i \in \mathcal{I}} P_{ik}$, where $P_{ik} = \frac{\exp(u_{ik})}{\sum_{j \in \mathcal{K}} \exp(u_{ij})}$ is the probability that i votes for k. The first-order condition for t_k is

$$0 = \frac{\partial V_k}{\partial t_k} = \sum_{i \in I} \frac{\partial P_{ik}}{\partial t_k} = \sum_{i \in I} P_{ik}(1 - P_{ik}) \frac{\partial u_{ik}}{\partial t_k} = -2A \sum_{i \in I} P_{ik}(1 - P_{ik})(\alpha_k^{s_{ik}} + t_k + \beta_k^{s_{ik}} x_i - x_i),$$

where $s_{ik} = A$ if $v_i = k$ and $s_{ik} = C$ otherwise. We obtain

$$t_k = \frac{\sum_{i \in I} P_{ik}(1 - P_{ik})(x_i - \alpha_k^{s_{ik}} - \beta_k^{s_{ik}} x_i)}{\sum_{i \in I} P_{ik}(1 - P_{ik})}. \tag{1}$$

The algorithm to estimate the equilibrium is as follows. We start with $v_i = v_i^0$ for each $i \in \mathcal{I}$. In each step we update t_k for each $k \in \mathcal{K}$ using Equation 1, and then update v_i based on the new utilities u_{ik}.

III. A reparametrization to describe the implications of the model

Before we had $A_k(x) = \alpha_k^A + \beta_k^A x$ and $C_k(x) = \alpha_k^C + \beta_k^C x$. Here we keep the linear functional form, but employ the following reparametrization. We define $p_k^A := \frac{\alpha_k^A}{1-\beta_k^A}$ and $p_k^C := \frac{\alpha_k^C}{1-\beta_k^C}$. We also re-label β_k^A, β_k^C as $1-\lambda_k$ and $1-\gamma_k$ because it de-clutters the notation and simplifies the formulas. Thus, the formulas for assimilation and contrast become $A_k(x) = (1-\lambda_k)x + \lambda_k p_k$ and $C_k(x) = (1-\gamma_k)x + \gamma_k p_k$, where the assumption now is that $\lambda_k < 1$ and $\gamma_k > 1$.

Conceptually, we can think of the parametrization as follows. Suppose for a moment that $p_k^A = p_k^C$. Then we can interpret $p_k = p_k^A$ as the "actual" position of the party. A k-supporter with position x sees the party closer to her, i.e., in the segment $\{(1-\lambda)x + \lambda p_k : \lambda \in [0,1]\}$. The smaller λ is, the closer to her; the closer λ is to 1, the less unbiased the perception is. Similarly, a non-supporter with position x sees the party farther from her, so she sees it in the same line, but outside the segment—this corresponds to $\lambda > 1$. See Figure 1 for a graphic illustration. Empirically we reject the equality $p_k^A = p_k^C$, so we can think of p_k^A and p_k^C as "anchors" (not the "true" position of the party) for k-supporters and non-supporters, respectively.

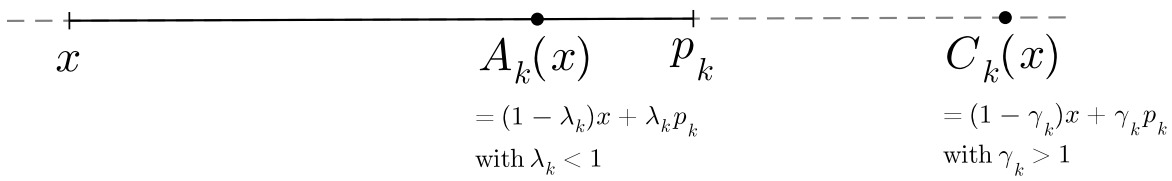


Figure 1: Illustration of how assimilation and contrast work under the alternative parametrization. Here $\lambda_k = .75$ and $\gamma_k = 1.5$, so a supporter perceives the party 25% closer to her, and a non-supporter perceives the party 50% farther from her, relative to where the anchor is.

IV. A voting model with assimilation and contrast under the reparametrization

There is a set $\mathcal{K}$ of parties. If a voter with ideology $x \in \mathbb{R}$ supports party $k \in \mathcal{K}$, she perceives the party's position as $A_k(x) = (1-\lambda_k)x + \lambda_k p_k^A$, where $\lambda_k \in (0,1]$ is the party's *assimilation* coefficient and $p_k^A \in \mathbb{R}$ is party k's "anchor" for supporters. If the voter does not support the party, she perceives the party's position as $C_k(x) = (1-\gamma_k)x + \gamma_k p_k^C$, where $\gamma_k \in [1,2)$ is the party's *contrast* coefficient. The lower λ_k and γ_k, the higher the party's valence advantage.

Each voter i supports a party $s_i \in \mathcal{K}$ at the start of the election campaign and has an ideological position $x_i \in \mathbb{R}$. Both are assumed to be fixed before the vote. Parties' parameters λ_k and γ_k are assumed to be fixed. Voters choose the party that they perceive to be closest to their ideological position. Parties choose and announce simultaneously a movement t_k in their position that induces the "anchors" p_k^A and p_k^C to move to $p_k^A + t_k$ and $p_k^C + t_k$ respectively. Their objective is to maximize the number of votes.

For simplicity, let's assume that there are only two parties, and let's call them D and R. A D-supporter with ideology x votes for D iff

$$-|A_D(x) - x| \geq -|C_R(x) - x|,$$

i.e., iff $\lambda_D|p_D^A + t_D - x| \leq \gamma_R|p_R^C + t_C - x|$, which happens iff x is not between

$$\frac{\gamma_R(p_R^C + t_R) + \lambda_D(p_D^A + t_D)}{\gamma_R + \lambda_D} \quad \text{and} \quad \frac{\gamma_R(p_R^C + t_R) - \lambda_D(p_{DA} + t_D)}{\gamma_R - \lambda_D}.$$

Let $\mu_D \in (0, 1)$ be the proportion of voters who are D-supporters, and let F_k be the CDF of k-supporters' ideologies, for $k = D, R$. Then D's vote share is

$$V_D(p_D, p_R) = \mu_D \left[1 - \left|F_D\left(\frac{\gamma_R(p_R^C + t_R) - \lambda_D(p_D^A + t_D)}{\gamma_R - \lambda_D}\right) - F_D\left(\frac{\gamma_R(p_R^C + t_R) + \lambda_D(p_D^A + t_D)}{\gamma_R + \lambda_D}\right)\right|\right] + (1 - \mu_D)\left|F_R\left(\frac{\gamma_D(p_D^C + t_D) + \lambda_R(p_R^A + t_R)}{\gamma_D + \lambda_R}\right) - F_R\left(\frac{\gamma_D(p_D^C + t_D) - \lambda_R(p_R^A + t_R)}{\gamma_D - \lambda_R}\right)\right|,$$

and R's vote share is $V_R = 1 - V_D$. Movements t_D, t_R constitute a *Nash equilibrium* iff t_D maximizes $V_D(\cdot, t_R)$ and t_R maximizes $V_R(t_D, \cdot)$.

Example: normal distribution of ideologies.—Assume that $p_k^A = p_k^C = p_k$ and k-supporters' ideologies follow normal distributions $N(x_k, \sigma_k^2)$ for $k = D, R$. In general there is no pure strategy Nash equilibrium. See Figure 2 for an example.

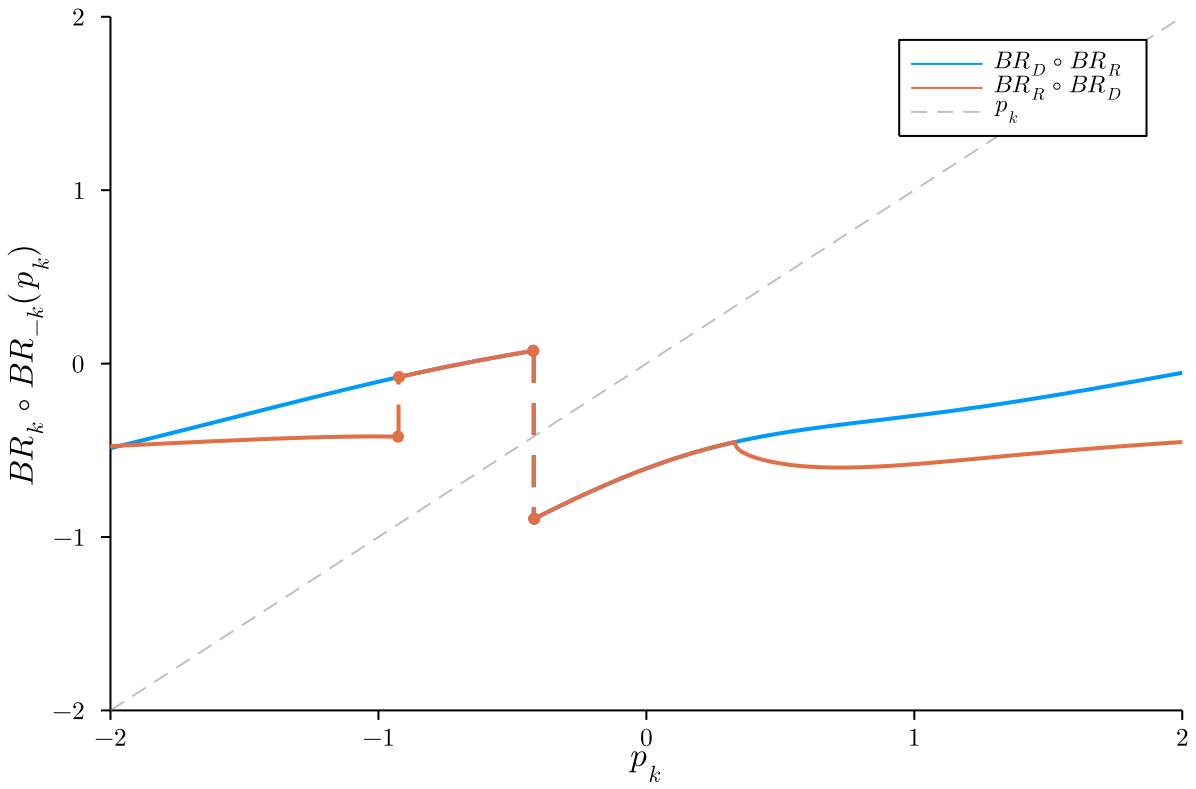


Figure 2: Best response to best response curves for each party in the deterministic model with $\mu_D = .5$, $F_D \sim N(-.5, 1)$, $F_R \sim N(.5, 1)$, $\lambda_D = .75$, $\gamma_D = 1.25$, $\lambda_R = .5$, $\gamma_R = 1.5$. The grey dashed line is a 45° line. A pure-strategy Nash equilibrium occurs when the best response to best response curves intersect the 45° line. We see that it does not happen in this model, since the curves move discontinuously from above the 45° line to below it, without crossing it. Thus, there is no pure-strategy Nash equilibrium in this game.

For a tractable special case that has a pure strategy equilibrium, assume $\mu_D = \mu_R$, $\sigma_D = \sigma_R$, $\lambda_D = \lambda_R$, $\gamma_D = \gamma_R$ and $x_R = -x_D$. Then

$$p_D = -\frac{\gamma^2 - \lambda^2}{\gamma^2 + \lambda^2} x_D \quad \text{and} \quad p_R = -\frac{\gamma^2 - \lambda^2}{\gamma^2 + \lambda^2} x_R$$

constitute an equilibrium. Note that there is divergence and that the leftist party strategically positions to the right of center (and vice-versa). We can derive comparative statics: more assimilation (low λ) and more contrast (high γ) entail more polarization. But this case is too special.

A. *Probabilistic voting*

Suppose that if a k-supporter i with ideology x_i votes for k she receives utility $-|A_k(x_i) - x_i| + \epsilon_{ik}$, where ϵ_{ik} follows some distribution symmetric around 0 with CDF G, and if she votes for $j \neq k$ she receives utility $-|C_j(x_i) - x_i|$. Then the probability that she votes for k is

$$\Pr(\epsilon_{ik} \geq |A_k(x_i) - x_i| - |C_j(x_i) - x_i|) = G\left(-\lambda_k |p_k^A + t_k - x_i| + \gamma_j |p_j^C + t_j - x_i|\right).$$

Therefore D's expected vote share is

$$V_D = \mu_D \int G\left(-\lambda_D|p_D^A + t_D - x| + \gamma_R|p_R^C + t_R - x|\right) dF_D(x) \\ + (1-\mu_D)\int G\left(-\gamma_D|p_D^C + t_D - x| + \lambda_R|p_R^A + t_R - x|\right) dF_R(x).$$

For tractability, let's assume $\epsilon_{ik} \sim \mathcal{U}[-\frac{1}{2\phi}, \frac{1}{2\phi}]$ with $\phi > 0$ small enough. Therefore D's expected vote share is

$$V_D = \mu_D \int \left\{\frac{1}{2} + \phi\left(-\lambda_D|p_D^A + t_D - x| + \gamma_R|p_R^C + t_R - x|\right)\right\} dF_D(x) \\ + (1-\mu_D)\int \left\{\frac{1}{2} + \phi\left(-\gamma_D|p_D^C + t_D - x| + \lambda_R|p_R^A + t_R - x|\right)\right\} dF_R(x).$$

Again, for tractability, let's assume that the distribution of k-supporters' ideologies is $\mathcal{U}[\underline{x}_k, \overline{x}_k]$ for $k = D, R$, and let $\Delta x_k := \overline{x}_k - \underline{x}_k$, and $x_k := \frac{\overline{x}_k + \underline{x}_k}{2}$. Assuming that $p_D^A + t_D, p_D^C + t_D, p_R^A + t_R, p_R^C + t_R \in [\underline{x}_D, \overline{x}_D] \cap [\underline{x}_R, \overline{x}_R]$, we have

$$\begin{aligned} V_D &= \frac{\mu_D}{\Delta x_D}\int_{\underline{x}_D}^{\overline{x}_D} \left\{\frac{1}{2} + \phi\left(-\lambda_D|p_D^A + t_D - x| + \gamma_R|p_R^C + t_R - x|\right)\right\} dx \\ &\quad + \frac{1-\mu_D}{\Delta x_R}\int_{\underline{x}_R}^{\overline{x}_R} \left\{\frac{1}{2} + \phi\left(-\gamma_D|p_D^C + t_D - x| + \lambda_R|p_R^A + t_R - x|\right)\right\} dx \\ &= \frac{1}{2} + \frac{1}{2}\phi\frac{\mu_D}{\Delta x_D}\left[-\lambda_D(p_D^A + t_D - \underline{x}_D)^2 - \lambda_D(p_D^A + t_D - \overline{x}_D)^2 + \gamma_R(p_R^C + t_R - \underline{x}_D)^2 + \gamma_R(p_R^C + t_R - \overline{x}_D)^2\right] \\ &\quad + \frac{1}{2}\phi\frac{1-\mu_D}{\Delta x_R}\left[-\gamma_D(p_D^C + t_D - \underline{x}_R)^2 - \gamma_D(p_D^C + t_D - \overline{x}_R)^2 + \lambda_R(p_R^A + t_R - \underline{x}_R)^2 + \lambda_R(p_R^A + t_R - \overline{x}_R)^2\right] \end{aligned}$$

and, of course, $V_R = 1 - V_D$.

In an interior Nash equilibrium we have $\frac{\partial V_D}{\partial p_D} = \frac{\partial V_R}{\partial p_R} = 0$, and solving for p_D and p_R we obtain

$$t_D = \frac{\frac{\mu_D\lambda_D}{\Delta x_D}(x_D - p_D^A) + \frac{(1-\mu_D)\gamma_D}{\Delta x_R}(x_R - p_R^C)}{\frac{\mu_D\lambda_D}{\Delta x_D} + \frac{(1-\mu_D)\gamma_D}{\Delta x_R}} \quad \text{and} \quad p_R = \frac{\frac{(1-\mu_D)\lambda_R}{\Delta x_R}(x_R - p_R^A) + \frac{\mu_D\gamma_R}{\Delta x_D}(x_D - p_D^C)}{\frac{(1-\mu_D)\lambda_R}{\Delta x_R} + \frac{\mu_D\gamma_R}{\Delta x_D}}. \tag{2}$$

The equilibrium movement of the parties are weighted averages of differences between the median supporter of each party and the anchor for them. We can derive comparative statics from these formulas. Less contrast (low γ_k), more supporters (high μ_k) and more concentrated supporters (low Δx_k) push the party more towards her supporters, while more assimilation (low λ_k) pushes the party more towards the other party's supporters. See Figure 3 for an example showing the comparative statics.

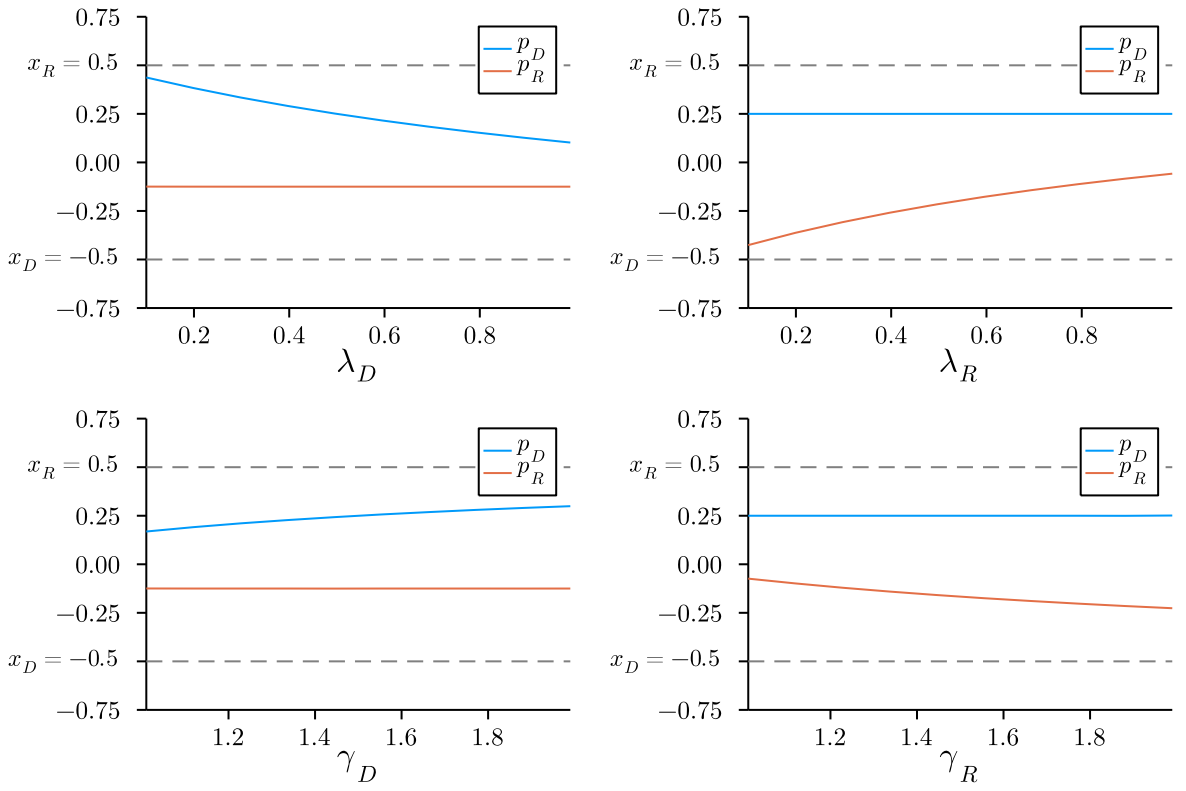


Figure 3: Equilibrium positions p_D, p_R in the probabilistic model with $\mu_D = .5$, $F_D = \mathcal{U}[-1.5, 0.5]$, $F_R = \mathcal{U}[-0.5, 1.5]$, $\lambda_D = .5$, $\lambda_R = .75$, $\gamma_D = 1.5$, $\gamma_R = 1.25$, $\phi = .1$, assuming $p_k^A = p_k^C$ for each $k = D, R$. The grey dashed lines are the position of the median supporter of each party.

Conceptually, there are various types of non-positional advantages in this model: the amount of (pre-campaign) supporters μ_k, the party's assimilation and contrast intensities (λ_k and γ_k), and the other party's assimilation and contrast intensities. They don't have the same effect on the equilibrium positions of the parties. Less contrast (low γ_k), and more (high μ_k) and more concentrated (low Δx_k) supporters push the party towards her median supporter (x_k), while more assimilation (low λ_k) pushes the party to compromise (i.e., to move closer to the other party's median supporter). The assimilation and contrast intensities of the other party do not have an effect on the position of the party.[1]

Take $k = D$. The intuition is that more assimilation (low λ_D) makes it easier to satisfy the party's supporters, so the party can cater to the other party's supporters (move towards x_R) without losing many votes. Less contrast (low γ_D) means that it is easier to attract R-supporters, which makes it less valuable to compromise ideologically. Having more supporters (high μ_D) means that catering to them is more profitable, which induces both parties to move to x_D. Having more concentrated supporters (low Δx_D) means that a small change in position can induce a large shift in D-supporters' vote (they are collectively more responsive), which induces parties

[1]This result follows from the non-substantive assumption that ϵ_{ik} is uniformly distributed. With other distributions, λ_{-k} and γ_{-k} would have an effect on the equilibrium p_k, although it's not clear in what direction.

to cater more to them.

Perception of the parties' positions.—The average perception of D is

$$\begin{aligned}
&\mu_D \int A_k(x)\, dF_D(x) + (1-\mu_D) \int C_k(x)\, dF_R(x) \\
&= \mu_D \int \left[(1-\lambda_D)x + \lambda_D(p_D^A + t_D)\right]\, dF_D(x) + (1-\mu_D) \int \left[(1-\gamma_D)x + \gamma_D(p_D^C + t_D)\right]\, dF_R(x) \\
&= \mu_D \left[(1-\lambda_D)x_D + \lambda_D(p_D^A + t_D)\right] + (1-\mu_D) \left[(1-\gamma_D)x_R + \gamma_D(p_D^C + t_D)\right].
\end{aligned}$$

Similarly, the average perception of R is

$$\mu_R \left[(1-\lambda_R)x_R + \lambda_R(p_R^A + t_R)\right] + (1-\mu_R) \left[(1-\gamma_R)x_D + \gamma_R(p_R^C + t_R)\right].$$

Using the equations for the equilibrium positions in the model with uniformly-distributed supporters (2) we obtain the the average perception of D's position is

$$\mu_D x_D + \mu_R x_R + \frac{\Delta x_R - \Delta x_D}{\frac{\Delta x_D}{\mu_D \lambda_D} + \frac{\Delta x_R}{\mu_R \gamma_D}} \left[(x_D - p_D^A) - (x_R - p_D^C)\right].$$

The last term indicates the deviation of the party's perceived position from the center $\mu_D x_D + \mu_R x_D$ of the voter distribution. Generically there is no convergence to the center.

V. ANES Presidential Election Figures

Figure 4: A-C Models by Election

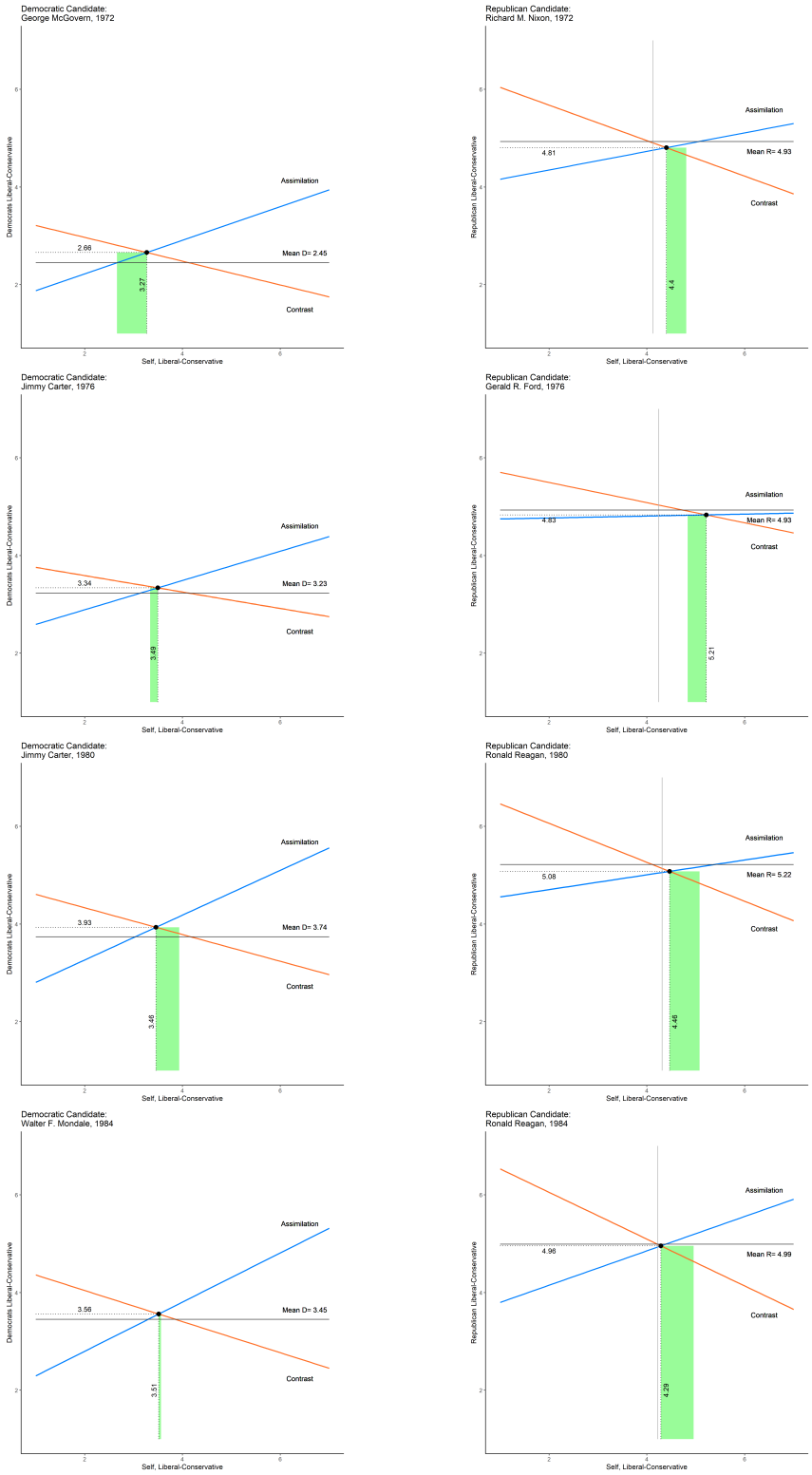


Figure 5: A-C Models by Election

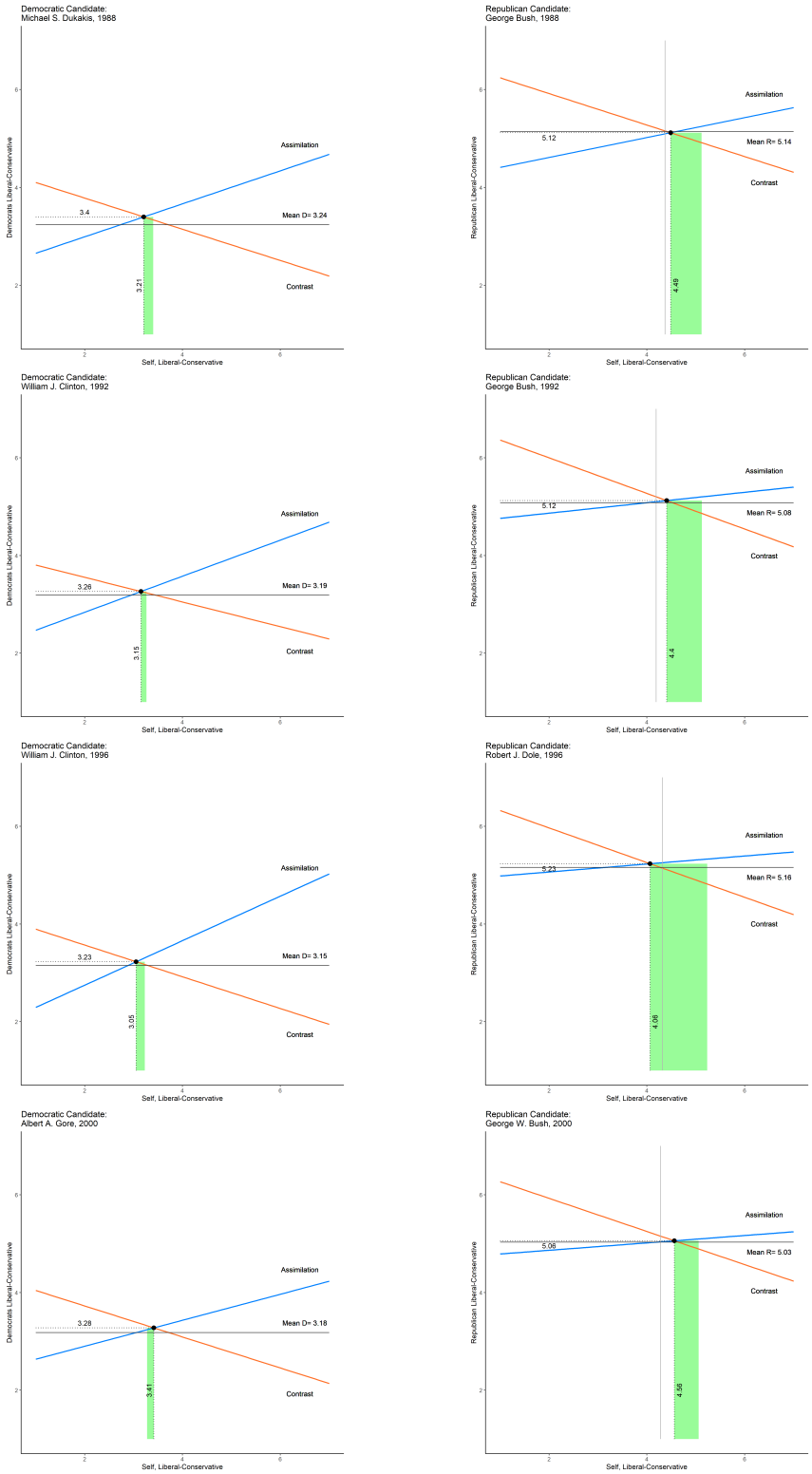
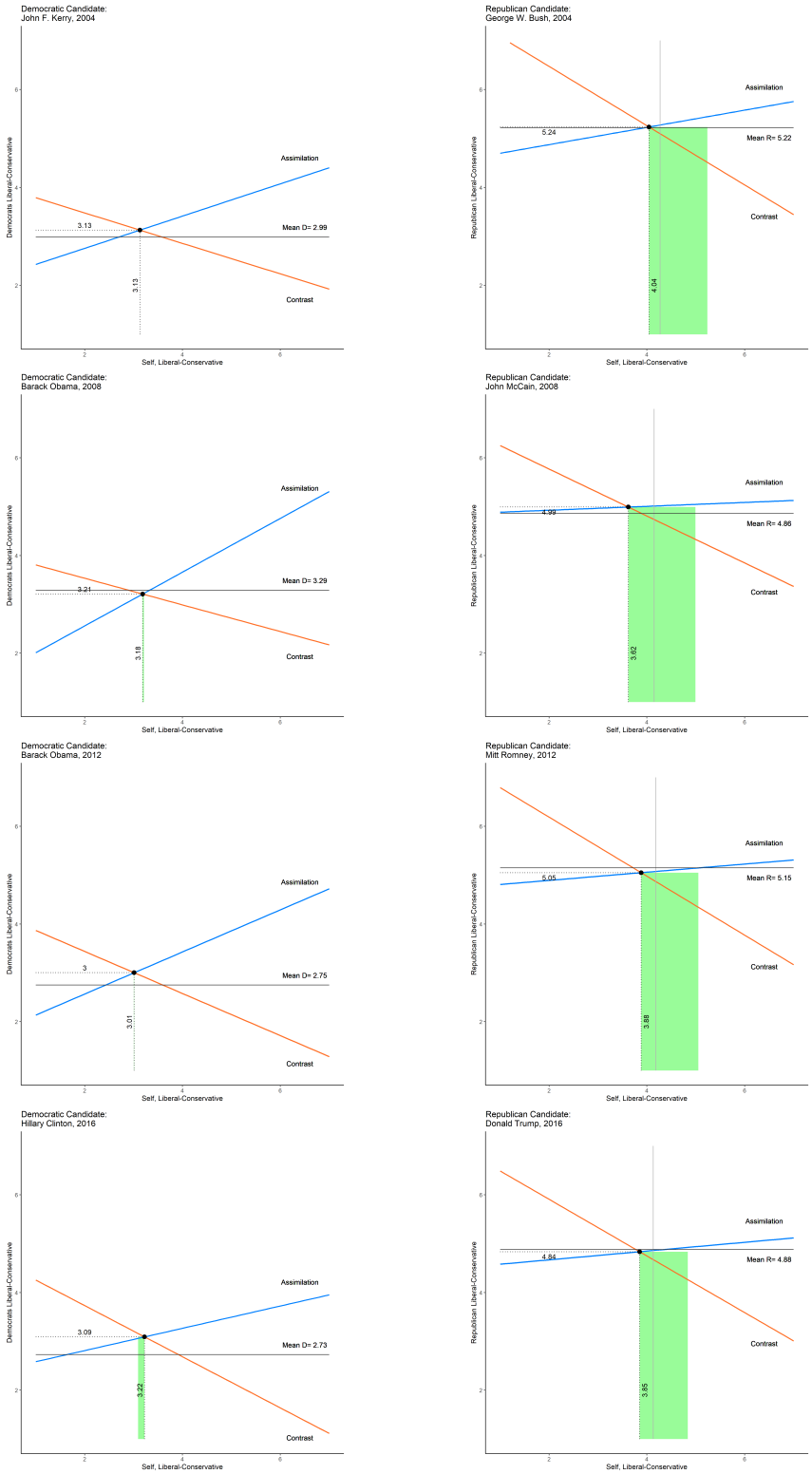


Figure 6: A-C Models by Election



VI. Estimates of Assimilation and Contrast for the Argentine 2020 Chequeado Survey (Figure 10)

Table 1: Argentina, 2020 Chequeado Survey

	Contrast Fernandez	Assimilation Fernandez	Contrast Macri	Assimilation Macri
	$C^D(x^R)$	$A^D(x^D)$	$C^R(x^D)$	$A^R(x^R)$
self-β	-0.383^{***}	0.539^{***}	-0.621^{***}	0.498^{***}
	(0.034)	(0.034)	(0.035)	(0.040)
Constant-α	-0.252^{***}	-0.792^{***}	0.854^{***}	-0.082
	(0.092)	(0.106)	(0.101)	(0.109)
N	862	636	972	559
Adjusted R^2	0.126	0.286	0.244	0.221
F Statistic	124.965^{***} (df = 1; 860)	254.903^{***} (df = 1; 634)	314.515^{***} (df = 1; 970)	159.023^{***} (df = 1; 557)

Notes:

***Significant at the 1 percent level.
**Significant at the 5 percent level.
*Significant at the 10 percent level.